METHOD 525.2

TIME WAVEFORM REPLICATION

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METHOD 525.2

TIME WAVEFORM REPLICATION

NOTE: Tailoring is required. Select methods, procedures, and parameter levels based on the tailoring process described in Part One, paragraph 4.2.2, and Annex C. Apply the general guidelines for laboratory test methods described in Part One, paragraph 5 of this Standard.

1. SCOPE.

1.1 Purpose.

Replication of a time trace under Time Waveform Replication (TWR) methodology in the laboratory is performed to:

a. Provide a degree of confidence that the materiel can structurally and functionally withstand the measured or analytically specified test time trace(s) to which the materiel is likely to be exposed in the operational field environment.

b. Experimentally estimate the materiel's fragility level in relation to form, level, duration, or repeated application of the test time trace(s).

1.2 Application.

1.2.1 Time Waveform Replication.

This test Method discusses TWR from a single-exciter/single-axis (SESA) perspective. Multiple-exciter TWR applications are addressed in Method 527.2. This Method provides guidelines for developing test tolerance criteria for single axis TWR testing. Annex A addresses SESA TWR testing by illustration. Annex B provides an overview of post-test analysis tools useful in TWR for verification of test tolerance compliance.

1.2.2 SESA Time Waveform Replication.

SESA TWR consists of the replication of either measured or analytically specified time trace(s) in the laboratory with a single exciter in a single axis, and is performed to accurately preserve the spectral and temporal characteristics of the measured environment. Without loss of generality in the discussion to follow, application of this Method will consist of a single time trace. SESA TWR in this Method is founded upon a "Deterministic/Probabilistic" framework of random process theory. An analytically specified time trace is assumed to be fully deterministic in nature with no relationship to a probabilistic framework, e.g., a chance of occurrence. A single measured time trace within a probabilistic framework is assumed to be a sample realization from an ensemble of possible time traces generated by an experiment that is replicated a number of times under identical conditions. For a single measured time trace, it is optimal to assume that the measured time trace represents the random process ensemble mean determined by averaging over an ensemble of records at each time increment, and has a confidence coefficient of 0.50. For more than one measured time trace captured under identical experimental conditions, it may be possible to create a time trace ensemble for which averaging over the ensemble members for each sample time increment yields valid estimates of the statistical moments for the unknown stochastic process underlying the time trace generation. This general deterministic/probabilistic philosophy for SESA TWR has important implications for time trace scaling considerations. Replicating a single time trace in this Method is generally transparent to the distinction between a deterministic time trace and the ensemble mean of a stochastic time trace.

Until recently, the replication of time traces representing measured samples of field environments varying in time and even frequency, or a combination of both time/frequency variations, was not possible using commonly available exciter control system software. The advent of more powerful data processing hardware/software, and the implementation of advanced control strategies, has led to exciter control system hardware and software that permit convenient replication of extended time-varying test environments on a single exciter in a single axis in the laboratory. TWR test methodology strongly reflects the concept of "test tailoring".

1.2.3 Time Trace.

The general term "time trace" is employed throughout this Method in an attempt to capture all of the possibilities of TWR applied in the replication of field measured (stochastic) or analytically specified (deterministic) environments in the laboratory. The following six forms of time trace are potential candidates for TWR testing.

a. Stationary random Gaussian time trace with arbitrary ASD of arbitrary duration.

b. Stationary random non-Gaussian time trace (for certain forms of non-Gaussian distribution, e.g., local skewness and high kurtosis) with specified ASD of arbitrary duration.

c. Short duration shock time trace.

d. Non-stationary time trace that has time-varying amplitude, time-varying frequency or both of an intermediate duration (longer than a typical shock time trace).

e. Non-stationary/stationary time trace that is repetitive at fixed period (e.g., gunfire shock).

f. Non-linear form time trace.

For general application, the time trace to be replicated under TWR is of a substantially shorter duration than typical stationary random environments, and usually of a longer duration than mechanical shocks. A TWR time trace may be composed of any combination of form specified in 1.2.3a through f above.

1.2.4 General Considerations and Terminology.

For purposes of discussion to follow, a single measured time trace is a function of finite duration having a uniform time sample increment and varying amplitude that is provided in digital form. For convenience, the single time trace under consideration is taken as acceleration, but the principles below apply equally well to other time trace representations such as velocity, displacement, force, etc.

It is assumed that for any measured physical phenomenon, the measurement can be repeated an indefinite number of times under the exact same conditions limited only by measurement resources, i.e., the underlying random process has an ensemble representation generally unknown. In the discussion to follow, reference to a measured time trace ensemble related to an underlying random process will assume the following:

a. Measured time traces are from a single physical phenomenon and have a joint correlation structure. This basically assumes a uniform and identical sample rate for all time traces, and common beginning and ending points.

b. The underlying random process has a deterministic component (or "signal") that can be estimated by the time-varying mean of the ensemble.

c. The underlying random process has a random component (or "noise") that can be estimated by a time-varying standard deviation of the ensemble.

d. If the measured time trace ensemble has only one member then this member will assume to be the underlying random process deterministic component or mean with a confidence coefficient of 0.5, i.e., this sample time trace has a 0.5 probability of being greater or less than the true underlying random process mean at each time increment.

> **NOTE**: This is not strictly correct because time traces have serial correlation information that essentially correlates the time trace from one time increment to the next time increment and, thus, the confidence coefficient may vary depending upon the degree of serial correlation.

Figure 525.2-1 provides a schematic outlining three basic TWR test modes designed to clarify the issue of time trace scaling. ***Generally, Method 525.2 attempts to define time trace scaling, but provides no direct guidance on time trace scaling; relegating the rationale for any time trace scaling to procedures outside this Method.*** The first TWR test mode involves a single measured time trace (or concatenation of N measured time traces) replicated under TWR with no scaling and no basis for scaling (termed ***NS*** for No-Scaling). In this mode there is no explicit ensemble basis for an underlying random process, and the time trace for replication is assumed to have a confidence coefficient of 0.50. A second mode for testing involves an ensemble of N measured time traces from a single phenomenon representative of sample functions from an underlying random process. In this second mode, any basis for scaling must be obtained from the N member ensemble, external to this Method, and will generally involve separate scale factors for the deterministic and random component estimates defined by the ensemble (termed ***ES*** for possibility of Ensemble-Scaling). A third mode involves an analytically specified time trace that assumes a basis for amplitude scaling (for a single time trace or an ensemble), and is termed ***AS*** for Analytical-Scaling. In this third mode the basis for scaling must come from outside this Method, and is generally "ad hoc" as will be defined in paragraph 1.2.6. A fourth mode of scaling with the intent of adding conservatism is possible through the introduction of increased test duration, and is termed as ***TS*** for Time-Scaling. In summary, (1) ***NS*** is the recommended fully tailored TWR testing that this Method is designed to address with no scaling allowed; (2) ***ES*** implies a proper mode of scaling based upon adequate ensemble sample trace information and rationale outside this Method, and (3) ***AS*** implies TWR testing using scaling based upon methodology outside this Method, but is not generally recommended unless the methodology has been properly validated. (4) ***TS*** implies conservatism in terms of test durations exceeding the basic mission scenario.

Scaling based upon other than measured ensemble statistics is termed ***ad hoc*** in this Method. As implied above, the creation of an ensemble implies that there exists an ensemble mean (deterministic component) estimate for the underlying random process, and a "residual ensemble" created by subtracting the mean from each member of the ensemble (random component) for the underlying random process. The deterministic component is "orthogonal" or uncorrelated to the random component by definition. Scaling for a measured ensemble based random process must consider individual scaling of both the deterministic and random components. Scaling based upon extraction of parameters from individual time traces, assessing these parameters, and scaling time traces based upon this parameter assessment in general is ad hoc. It is termed "ad hoc" because it scales the deterministic component and the random component essentially the same. For such ensemble representation, the deterministic component (the signal) and the random component (the noise) need to be scaled separately.

Underlying random processes within this Method will be assumed to have sampled continuous time traces e.g., analog voltage signal, in contrast to discrete processes such as a Poisson counting process trace. However, a laboratory test scenario may incorporate a discrete underlying random process through application of a series of concatenated time traces under TWR. Such an extended laboratory test scenario may provide more overall information for materiel structural and functional integrity assessment. Extended laboratory test scenarios will be discussed further when test axes, duration, and the number of time trace(s) applications are discussed in paragraph 2.3 below. It would also appear that TWR is capable of replication of time traces that are generated as result of reducing a uniformly sampled time trace for fatigue purposes. Typically, traces suitable for fatigue testing only consist of discrete peak and valley points, and are the result of applying a cycle counting process to a uniformly sampled time trace. Cycle counting and peak/valley identification generally distort the measured time trace in time, and can be characterized as a form of nonlinear time trace that can be forced to be band-limited within the exciter bandwidth through appropriate interpolation.

1.2.5 Time-Varying Time Trace - Physical Phenomenon.

A time-varying trace captured in measurement signals is caused by the time-varying phenomenon that is being measured. In general, the time-varying characteristics of the environment (excluding shock) are longer than the lowest resonant frequency characteristics of the materiel under test. In particular, a time-varying trace may range from three seconds to several hundred seconds.

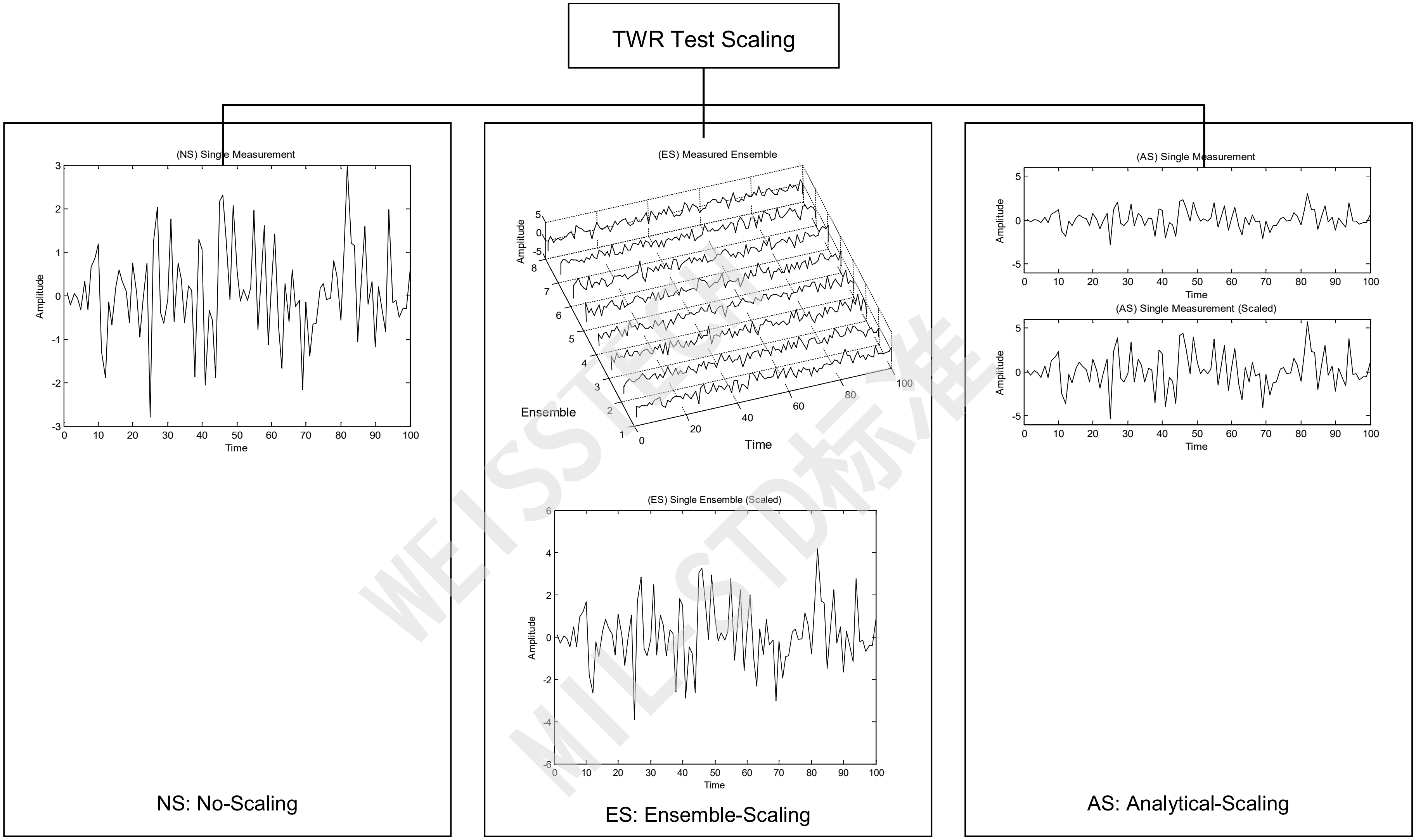

Figure 525.2-1. Basic TWR test modes as related to time trace scaling.

1.2.6 General TWR Test Philosophy With Regard To Time Trace Simulation (and Scaling).

As emphasized in paragraph 1.2.4, time trace scaling to enhance conservativeness of laboratory testing is generally outside the scope of this Method. Figure 525.2-2 defines simulation possibilities within TWR including time trace scale rationale assumed to be provided external to this Method.

Two terms important to understanding TWR simulation will be introduced. The first term, ***intrinsic statistics***, refers to the time-varying statistical estimates available from a single measured time trace (generally from short-time estimates). A single time trace has a confidence coefficient of 0.50, and the time-varying statistical estimates provide no information relative to the underlying ensemble-based random process, except for an estimate of the mean of the underlying random process. The second term, ***extrinsic statistics,*** refers to the time-varying statistical estimates available from more than one measured time trace, which forms a sample time trace ensemble. In this case, not only is an estimate of the underlying random process mean available, but also an estimate of its variance on a time increment basis. For comprehensive LCEP directed TWR materiel testing specifying analytical time traces through simulation, knowledge of the extrinsic statistics is essential. In general, specifying analytical time traces through simulation based upon intrinsic statistics is very limited, and usually unreliable for testing to the underlying random process (Method 519.8, Annex B discusses this further). Conversely, if a very small measured time trace sample ensemble is available, estimates of the underlying random process parameters tend to have large errors providing for an unreliable simulation. In this latter case, a more optimum test scenario is provided by replication of each of the individual measured time traces in a pre-defined sequence. A useful way to view intrinsic versus extrinsic statistics is to envision a One-Way Analysis of Variance, whereby the intrinsic statistics correspond to the “error within”, and the extrinsic statistics correspond to the “error among”.

Figure 525.2-2 attempts to clarify simulation issues for the four potential TWR test modes provided in the Figure. Whenever simulation is undertaken, it is implicit that the measured time trace(s) is scaled as a result of the simulation. This scaling is not considered “ad hoc” per se. The left most portion of the figure provides the simplest TWR test scenario with a single measured time trace and no scaling ***NS*** and no simulation (termed ***SM*** for Single-Measured). The left center portion of the figure provides for a single measured time trace with intrinsic trace time-average estimation used for creation of a simulated ensemble consisting of a single time trace, where ***AS*** is implied (termed ***SS*** for Single-Simulated). The right center portion provides the case of multiple measurements from a single phenomenon, with ensemble creation followed by simulation based upon combined intrinsic/extrinsic statistics and ***ES*** implied (termed ***MS*** for Multiple-Scaled). The right-most portion of the figure provides the case of multiple measurements from a single phenomenon, and the possibility of concatenation of the measurements (assuming ensemble information for simulation is too limited) (termed ***MM*** for Multiple-Measured). For generality, ***MM*** may allow for (but does not recommend) the use of “ad hoc” scaling of the individual measurements to be concatenated. To summarize, (1) ***SM*** is the recommended basic fully tailored TWR testing that this Method is designed to address; (2) ***SS*** is a less desired approach to replication of details of a single time trace with a minimal set of information that implies scaling a single time trace; (3) ***MS*** is recommended as a specialized information/labor intensive, but faithful approach to replication of an underlying random process under TWR and, finally, (4) ***MM*** is recommended for a time trace concatenation form of testing where “ad hoc” scaling procedures are best not applied.

It is vitally important that the distinctions made in Figure 525.2-1 and Figure 525.2-2 be recognized in TWR testing. In addition it is important to note the following:

a. For zero mean Gaussian distributed stationary time traces, scaling is upon the random component alone, and ways of performing scaling for more than one time trace are provided in Method 519.8, Annex A. For these time traces, the statistics in the frequency domain, i.e., autospectral density estimates, are computed and envelopes determined.

b. For time traces with a time-varying mean-square, it is unlikely that the ensemble representation of the underlying random process will have a time invariant or constant variance. If the underlying random process has a time-varying variance, then the sample time traces cannot be scaled by a constant and still preserve the probabilistic structure of the process.

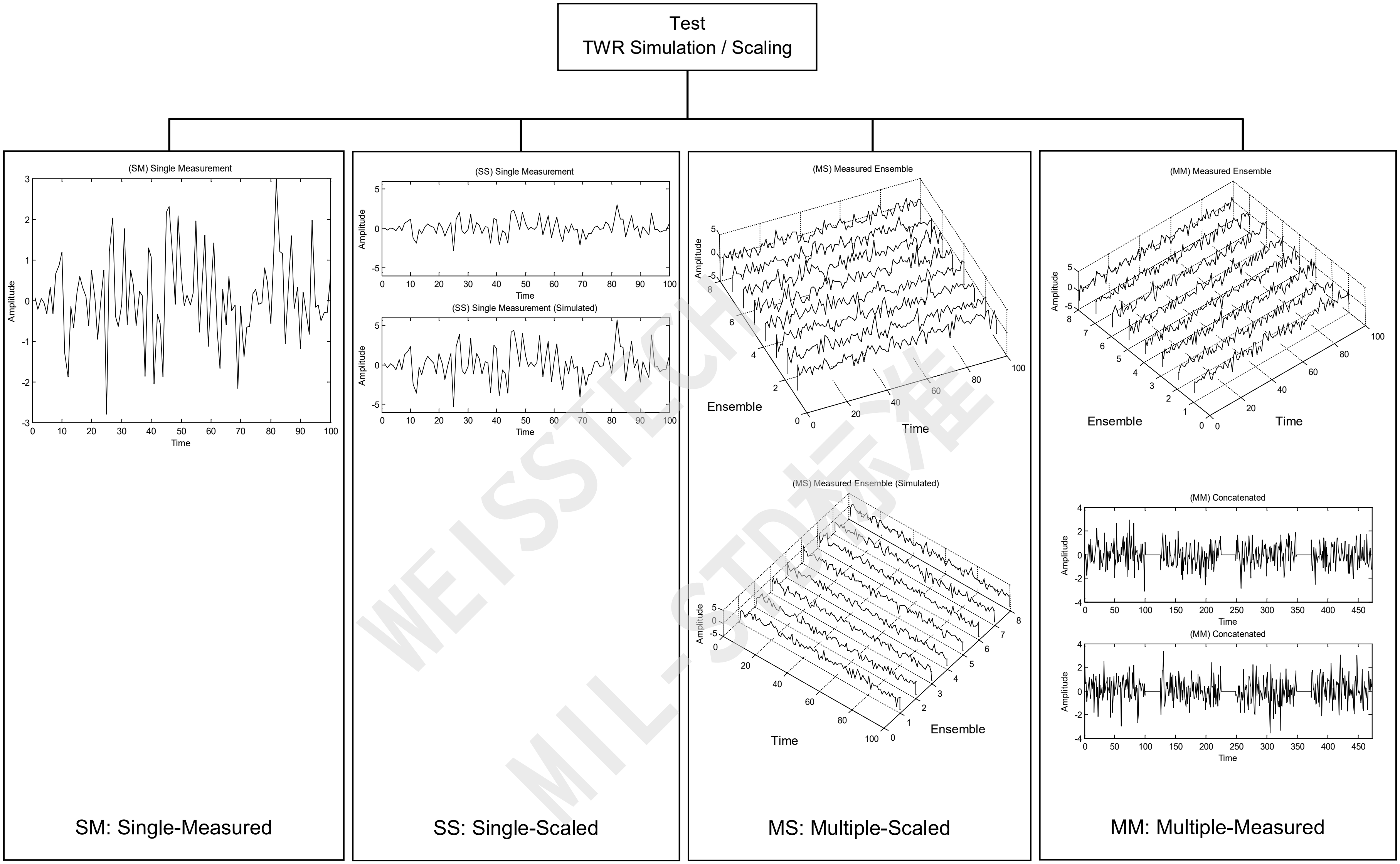

Figure 525.2-2. Basic TWR test simulation combinations.

c. For multiple time traces from the same underlying random process, creation of an ensemble may not be straight forward since it is nearly impossible to obtain measured time traces with exactly the same length by repeating the experiment, i.e., collection process (see paragraph 6.1, reference c.). It is also important to remember that the measured time traces must be "registered" or "serially correlated" according to some physical phenomenon, so that averaging over the ensemble members for each sample time point is meaningful. In the case where a ***valid ensemble*** is available, it is possible to estimate both the mean and variance of the random process at each time increment by averaging over the ensemble members. Under these circumstances, TWR testing could proceed on the basis of use of (a) the ensemble mean, (b) the "maximum" of the ensemble members, (c) all N ensemble members, or (d) the ensemble mean plus (minus) a proportion of the square root of the ensemble variance. All four of these choices will preserve the probability structure of the unknown random process underlying the ensemble realizations. It is vitally important to note that "scaling" the ensemble mean, or any ensemble member by a constant factor, in general, will not provide time traces that are representative of the probability structure of the random process, unless the variance of the unknown random process is constant in time. Use of (d) above for TWR testing needs further amplification. The variance estimate obtained from averaging over the ensemble at each time increment will provide an unbiased estimate of the variance at the time increment with substantial random error or variation. Scaling each time point by the square root of the variance (with appropriate sign) provides for a "non-linear" transformation of the scaled time trace (since adjacent time increments may be scaled by factors that are different by an order of magnitude). Thus it becomes necessary to smooth the ensemble variance estimate in time to obtain acceptable time-varying scale factors. This smoothing introduces bias error with the benefit of decreased random error or variability. Unfortunately, there is little concrete guidance on the degree of smoothing that should be applied and, in fact, this becomes a form of a non-linear regression problem (i.e., smoothing is dependent upon the true unknown shape of the data being smoothed). Scaling based upon statistical ensemble estimates should only be performed by a competent data analyst familiar with random process theory, and the techniques of non-linear regression.

This summarizes the rationale behind the philosophy of this Method of simulation, and not directly recommending the "scaling" of measured time traces. Method 519.8, Gunfire Shock, Annex B, discusses extensively scaling for measured gunfire time traces.

In TWR testing involving analytically-specified deterministic time trace information, there is substantial test flexibility depending upon the assumptions that are made, be they ad hoc or from some rational basis. In this case, this Method becomes merely a tool for replicating what is generated without regard for the assumptions behind the specification. Any rationale for scaling is again external to this Method.

1.3 Limitations.

This Method addresses very general time-varying traces not necessarily identifiable with underlying stationary or non-stationary random processes. It is apparent from various vendor TWR hardware/software configurations that the only requirement for application of this Method is the band-limited character of the time trace for replication, and its compatibility with the band-limited characteristics of the device (exciter) to be driven with the TWR hardware/software. For example, measured time traces that vary in frequency can be replicated as long as the time trace bandwidth is limited to overall bandwidth of the exciter control system. Non-Gaussian time traces can be replicated under TWR. All measured time traces can be replicated under TWR, provided they are within the band limit capabilities of the exciter control system to which they are applied for testing purposes. Limitations of this Method include the following:

a. Does not address very long (several hour) time traces that can be termed "stationary" in nature (Gaussian or non-Gaussian and possibly have significant discrete components e.g., UAV measured environments). It is possible to repeat a given time trace multiple times, however, variations associated with actual experiment repetitions in the field will not be captured. It is important to note that, given a single stationary Gaussian or non-Gaussian time trace of sufficient length, it is possible to (1) divide this time trace into multiple time trace segments at zero crossings (required close to zero mean for each segment) and, (2) randomly place these segments into a permuted order to generate multiple time traces of sufficient length but essentially "stochastically independent" of one another. This can be particularly attractive for measured stationary non-Gaussian environments where the non-Gaussian "exact moment structure" must be preserved over long periods of time. The alternative to this is precise modeling of the measurement time trace and subsequent stochastic generation of unlimited segments for TWR input.

b. Does not address the advantages and disadvantages of replicating very short duration time traces (shocks) over and above application of Method 516.8.

c. Does not explicitly address time traces that have highly variable frequency characteristics in time.

d. Does not explicitly address time traces that are nonlinear in nature.

e. Does not explicitly address repeated environments that may be of a non-stationary nature because of the occurrence pattern of the environment. For example, no discussion is provided on occurrence statistics that may be modeled in terms of a non-stationary (rate-varying) Poisson process.

f. Generally does not address the characteristics of the time trace on the materiel in terms of materiel "rise-time" response.

2. TAILORING GUIDANCE.

2.1 Selecting the TWR Method.

After examining requirements documents and applying the tailoring process in Part One of this Standard to determine where significant time-varying effects are foreseen in the life cycle of the materiel, use the following to confirm the need for this Method and to place it in sequence with other methods.

2.1.1 Effects of Transition To Time Trace TWR.

Method 525.2 is broadly consistent with the philosophy of test tailoring. A substantial high amplitude field measured time trace has the potential for producing adverse effects on all electronic materiel. The potential for adverse effects may be related to transition time and duration of the time trace. When transition to the time trace and time variation characteristics in the time trace is short, "rise times" in materiel response may be adequate to cause degradation in performance. When duration of the time trace is substantial in comparison to the transition times, the effects to materiel, e.g., low cycle fatigue, may also be substantial. In performing a TWR test, it is desirable that the onset/termination of the significant environment be consistent with the onset/termination of the environment anticipated in the field.

2.1.2 Sequence Among Other Methods.

a. General. Use the anticipated life cycle sequence of events as a general sequence guide (see Part One, paragraph 5.5).

b. Unique to this Method. Generally, significant time-varying traces may occur at any time during the life cycle of the materiel, and are usually interspersed among stationary random and shock environments that are covered under guidance provided in Methods 514.8 and 516.8, respectively.

2.2 Selecting a Procedure.

This Method includes two basic test procedures:

a. Procedure I: The SESA replication of a field measured materiel time trace input/response.

b. Procedure II: The SESA replication of an analytically specified materiel time trace input/response.

Based on the test data requirements, determine which test procedure is applicable. In particular, determine if there exists a carefully measured and properly processed field measured time trace, or if there is a generated, uniformly sampled band-limited analytical time trace. Determine if the time trace can be placed in an ASCII data file for archive and replication. If there are field measured or analytically specified environmental time traces for a materiel component, determine if the time trace(s) has an extended form over the entire materiel, i.e., determine the extent of spatial correlation.

2.3 Determine Test Levels and Conditions.

For TWR replication of measured time traces in the laboratory, the test levels are fully specified by the field measured time traces. If several field measured time traces are available, generally, the tester will want to make up a single ASCII file consisting of several "events" appropriately spaced in time. In general, for this Method, Procedure I, it is not recommended that any factor, constant or otherwise, be applied to "enhance" the measured time trace for testing (for reasons discussed in paragraph 1.2.6). For this Method, Procedure II, any scaling must be consistent with information in paragraph 1.2.6 and, generally, the scaling must not be ad hoc in nature. It is not recommended that time traces that exceed the capacity of the vibration exciter be scaled down by gain, e.g., run at –3 dB. For pretest exciter control system compensation, i.e., establishing the exciter system transfer function, the time trace may be applied at lower levels to either the test item or to a dynamically similar surrogate. Identify the test conditions, particularly with respect to temperature. Exercise extreme care in consideration of the details in the tailoring process. Base the test level and condition selections on the requirements documents, the Life Cycle Environmental Profile, and information provided within this procedure.

2.3.1 General Considerations.

As has been mentioned in paragraph 1.2, statistical estimates defining the behavior of a non-stationary random process can only be made on ensembles of time traces from the non-stationary process. Typically, only one sample time trace from an ensemble of an unknown non-stationary random process is available. It is absolutely essential that the test time trace be fully documented such that transfer of an ASCII file of the test time trace can be made to other laboratories for application or testing, and be repeated in the future. Information on the location of measurement transducers and general test configuration must accompany the test time trace. Any such analytical description can be tied directly to comparison between the time trace input to the exciter control system (reference time trace) and the test output as recorded by the exciter control system (control time trace). To clarify the terminology standard, the "reference time trace" is merely the ASCII representation of the time trace for the laboratory test. The "control time trace" is the ASCII digital file created by the exciter control system representing the "result" of the test. This control time trace is created by converting an analog voltage signal from a measurement device, e.g., an accelerometer mounted on the test item or test item interface at the location that the reference time trace is to be replicated, to a digital form by a signal conditioned analog-to-digital device. It is referred to as a "control" time trace because it is in the comparison of the reference time trace to the control time trace that the analog input to the exciter device is compensated in order to reproduce the reference time trace. The "control" time trace represents the "best fit" of the output of the exciter control system parameters through compensation to the desired input reference time trace. Annex A provides the details of a typical time reference/control comparison. A successful test under TWR is defined as a test, whereby the control time trace compares to the reference time trace within the tolerance limits specified for the test. The tolerance limits may be specified in the time domain, the frequency domain or a combination of the two. Annex B provides the basis for developing meaningful tolerance limits under SESA TWR. Rudimentary tolerance limits are provided within most vendor supplied TWR software for purposes of "controlling," i.e., appropriately compensating the system prior to test but, in general, the test laboratory will want to establish and implement some well-defined analytical procedures for comparing the control time trace ASCII file with the reference time trace ASCII file. Annexes A and B provide guidance in this area.

The test item may be instrumented at other locations than at the point of "control." The other measurements made during testing are referred to as monitoring measurements. Such measurements may be useful for purposes such as analytical modeling of the materiel, or just monitoring materiel response dynamic characteristics, and will not be discussed further here. For SESA exciter laboratory testing, the TWR software allows only single measurement comparison and monitoring for signal compensation "control" purposes.

For the TWR procedure, subject the test item to a sufficient number of suitable time trace events to meet the specified test conditions. Generally, the number of times the test item is subject to a given time trace event is determined from the materiel's life cycle profile in much the same way the duration for stationary random vibration is determined or the number of shock applications for shock is determined. In any case, subject the test item to no fewer than three time trace events for establishing confidence in the materiel's integrity under test if specific information from the materiel's life cycle profile is not available.

2.4 Test Item Operation.

Whenever practical, ensure the test item is active and operating during TWR testing. Monitor and record achieved performance correlated in time with the test time trace. Obtain as much data as possible that define the sensitivity of the materiel to the time trace environment. Where tests are conducted to determine operational capability while exposed to the environment, operate the test item. In other cases, operate the item where practical. Operation during transportation will not be possible in almost all cases. Also, there are cases where the operational configuration varies with mission phase, or where operation at high time trace levels may not be required, and may be likely to result in damage.

3. INFORMATION REQUIRED.

3.1 Pretest.

The following information is required to conduct and document TWR tests adequately. Tailor the lists to the specific circumstances, adding or deleting items as necessary.

a. General. Information listed in Part One, paragraphs 5.7 and 5.9; and Part One, Annex A, Task 405 of this Standard.

b. Specific to this Method.

(1) Test system (test item/platform configuration) detailed information including:

(a) Control sensor location for control time trace (for single axis testing this will be a point near the original reference measurement point).

(b) Reference time trace to be replicated (stored on the TWR control system disk).

(c) Monitor sensor locations (if any).

(d) Test bandwidth and preprocess reference time trace as required.

(e) Levels of pre-test acceptable to obtain appropriate exciter system compensation.

(f) Criteria for satisfaction of the test including TWR tolerance limits related to the reference time trace and the control time trace(s).

(2) Ability of overall system to replicate the time trace under TWR including band-limited input and the temperature effects (if any). For the application of more than one time trace, the individual time traces must be separated at time intervals that allow the test item to assume a pre-test dynamic condition (unless this is contrary to the requirements of the LCEP). Impedance mismatches and boundary conditions are important for assessing the ability to execute a successful TWR test.

c. Tailoring. Necessary variations in the basic test procedures to accommodate LCEP requirements and/or facility limitations.

3.2 During Test.

Collect the following information while conducting the test:

a. General. Information listed in Part One, paragraph 5.10; and in Part One, Annex A, Tasks 405 and 406 of this Standard.

b. Specific to this Method.

(1) Capture of the control time trace in digital form for comparison with the reference time trace.

(2) Capture of the monitor time traces in digital form.

(3) Recording of the number of individual test events and order for application.

(4) Log of auxiliary environmental conditions such as temperature.

(5) Log of materiel functional failure.

3.3 Post-Test.

The following post test data shall be included in the test report.

a. General. Information listed in Part One, paragraph 5.13, and in Annex A; Tasks 405 and 406 of this Standard.

b. Specific to this Method.

(1) Number of exposures of the test item to the time trace(s) and the order if several dissimilar time traces are used in test.

(2) Any data measurement anomalies, e.g., high instrumentation noise levels, loss of sensor response.

(3) Status of the test item/fixture. In particular, any structural or functional failure of the test item/fixture.

(4) Status of measurement system after each test.

(5) Any variations from the original test plan.

4. TEST PROCESS.

Tailor the following paragraphs, as appropriate for the individual contract or program.

4.1 Test Facility.

Use a test facility, including all auxiliary equipment, capable of executing the TWR test with the control strategies and tolerances discussed in paragraph 4.2. In addition, use measurement transducers, data recording, and data reduction equipment capable of measuring, recording, analyzing and displaying data sufficient to document the test and to acquire any additional data required. In particular, decide on the means of determining if test tolerances have been met through either vendor supplied measures or digital post-processing measures as described in the Annexes. For TWR testing it is important that all measurements and monitoring of test item functioning be correlated in time.

4.1.1 Procedure I - The SESA Replication of a Field Measured Materiel Time Trace Input/Response.

The SESA replication of a field measured time trace representing an input to the materiel or a response of the materiel considers only an un-scaled measured time trace in the laboratory with a single exciter in a single axis or mechanical degree-of-freedom.

4.1.2 Procedure II - The SESA Replication of an Analytically Specified Materiel Time Trace Input/Response.

The SESA replication of an analytically specified time trace representing an input to the materiel or a response of the materiel considers carefully scaled versions of a measured time trace in the laboratory with a single exciter in a single axis or mechanical degree-of-freedom.

4.2 Controls.

4.2.1 Calibration.

Ensure for the exciter system, all transducers, signal conditioning equipment, independent measurement systems, and the exciter control system hardware are calibrated for conformance with the specified test requirement(s). Ready access to the reference, control, and drive time trace files in ASCII form will be required for independent confirmation of adequacy of the time trace replication for a successful TWR test.

4.2.2 Tolerances.

a. General Philosophical Discussion. At this point in TWR test methodology, test tolerance specification is not well quantified. Test tolerance development for TWR is based upon a different laboratory test philosophy as opposed to the test philosophy contained in Methods 514.8 and 516.8. The reason for this change in philosophy is embedded in the implementation of TWR testing. TWR testing may involve replicating a combination of stationary Gaussian, stationary non-Gaussian, and nonstationary measured environments within a single time trace designated the reference time trace. Tolerance specification may be related to current tolerance specification in Methods 514.8 and 516.8, or be independently established based upon the nature of TWR testing. First, it is important to note that TWR does not provide a ***"waveform control strategy"*** that implies the satisfaction for the time control trace of each of the time/amplitude coordinates of every point within the reference time trace (satisfaction to within some predetermined amplitude tolerance, while totally satisfying the sampling time constraint). Exciter control and feedback hardware/software configurations to accomplish this to a bandwidth of 2000 Hz are currently not available. TWR implicitly "averages" the reference time trace (waveform) information over both time and frequency. There are two sources for the time and frequency averaging. The first source is through compensation of the voltage drive waveform by ***linear convolution*** of the exciter system impulse response function estimate with the reference time trace. The condition of system linearity is almost never satisfied so that the reference time trace is averaged over time through the linear convolution (as opposed to providing convolution through a two-dimensional non-stationary/nonlinear impulse response function that changes instantaneously in time). The second source is the implicit and nearly unavoidable averaging of significant amounts of energy from signals outside of the reference time trace bandwidth (i.e., the bandwidth for TWR control). These two sources of time/frequency averaging severely limit consideration of time point (or increment) by time point (or increment) amplitude tolerance limit specification between the reference and control time traces. Experience has shown that the distribution of the time point by time point difference between the reference and control time traces is almost always non-Gaussian distributed, leading to the need for a complex tolerance specification and interpretation. Even though this may seem to be a significant limitation for the implementation of TWR testing, it is important to realize that the focus of TWR is replication of a stochastic field environment for which any one measured sample time trace (out of a potentially infinite number of such traces) has a zero probability of occurrence. Because the exact probability structure of the ***"true"*** field environment is generally unknown, this implies that the test tolerance specification can be quite broad, and the objective of the test (be it structural integrity or functional capability) can be satisfied at the same time. In the broadest interpretation, this can border on concluding that if the reference and control time traces plotted side-by-side visually "look alike", then tolerance in terms of random process theory and sample functions has been met, even though the time-point by time-point amplitude (TPP) difference between the reference and control traces may be substantial. In the tolerance consideration for this Method, although TPP provides an interesting display by plotting the reference time trace versus the control time trace along orthogonal axes (see Annex A), it is not recommended that TPP comparison be the major determiner for test tolerance satisfaction. Instead, recommend that time and frequency average estimates made over the same time frame on the reference and control time traces be used for tolerance specification. In particular, it is recommended that frequency based averages incorporated into ASD, SRS estimation, and time-based averages incorporated into mean-square (or root-mean-square) estimation be used in tolerance specifications whenever possible. Methods 514.8 and 516.8 incorporate test tolerances on ASD and SRS estimates, respectively. The tolerances in these two methods are easily interpreted, and generally are easily satisfied in TWR testing. With regards to time based averages, it is important to note that while the root-mean-square of the difference between two independently distributed Gaussian random variables is a function of the square-root of the sum of their variances, the difference of the root-mean-square levels of the two random variables (averaged over a certain number of realizations) may be an order of magnitude or more less. That is, the variance of an average of N variables from a probability distribution with variance σ^2 is σ^2/N. Annexes A and B discuss the form for tolerance specification in more detail. In the paragraphs to follow, the term "Specialized Test Tolerance Requirements" (STTR) will be used. Use of STTR recognizes that TWR testing may require a level of sophistication in environmental test tailoring not experienced in the standard methods. For example, materiel exposed to high levels of kurtosis may require TWR test methodology based upon field measurements. Such a specialized laboratory test may require verification of the kurtosis levels, and a detailed specification of the shape of the probability density function to ensure other probability distribution moments are acceptable. It is not feasible in this Method to prescribe acceptable tolerance limits for this scenario. Thus, such tolerance limits will be developed under the term STTR and will require trained analysts for specification and interpretation. This allows the focus in paragraphs 4.2.2b and 4.2.2c of a more practical nature.

b. Practical Tolerance Considerations. Laboratory testing in another method that is implemented by using TWR test methodology should be under laboratory test tolerance requirements in the other method. For example, Method 516.8 provides tolerances on shock under the SRS methodology. For a measured shock time trace replicated under TWR test methodology, the same SRS based test tolerances should apply for comparison of the reference time trace SRS with the control time trace SRS. In general, tolerances specified for TWR test methodology should be ***consistent with, and no broader than*** laboratory test tolerances in other methods for testing with similar objectives. Relative to TWR test methodology on measured time traces of diverse form, measured mechanical response ***time traces*** and ***portions of such time traces*** may have any one of three characteristic forms.

(1) The first form is that of Gaussian or non-Gaussian stationary random vibration.

(2) The second form is that of a short duration high level transient or shock where the duration of the transient is much shorter than the periods of the lowest natural frequencies of interest for the materiel.

(3) The third form is that of a non-stationary transient vibration having duration that substantially exceeds the period of the lowest natural frequency of the materiel.

A fourth form, too specialized for consideration here, might be classed as periodic repetition of an event for which test tolerance is established according to time trace ensemble statistics (see Method 519.8, Gunfire Shock). For TWR tolerance development, such tolerances ***should not exceed*** the tolerances provided for stationary random vibration and mechanical shock for the first and second forms, respectively. It is anticipated that a properly designed TWR test will easily meet the tolerance levels specified in both of these forms (Methods 514.8 and 516.8). The tolerances for the third form of non-stationary time trace are somewhat dependent upon the nature of the non-stationarity. Techniques for non-stationarity assessment in which time trace amplitude is a function of both time and frequency are available (see paragraph 6.1 references a and b). Some non-stationary time traces that have time invariant frequency characteristics can be represented by the Product Model (PM), and can be processed for tolerance purposes as stationary random vibration with a time-varying envelope. Annexes A and B should be consulted for details of TWR tolerance specification for non-stationary time traces. If it is unclear as to how to segment a TWR time trace, then (1) time-average test tolerances may be provided on the difference between the control and reference time traces, or (2) digital bandpass filtering may be performed on both the control and reference time traces to make common bandwidth comparisons. The Annexes should be consulted for such tolerance development.

c. Tolerance Recommendations. In general, all test tolerances need to be established by some comparison in the time domain and frequency domain of the digitized reference and control time traces. Rudimentary comparison that might be taken for nominal test tolerances is usually performed by the vendor-supplied TWR software. The vendor will typically refer to the rudimentary comparison as “rms error.” Test laboratory personnel need to consult the vendor supplied TWR system manuals for such error considerations, and have a very clear understanding of the proper interpretation and meaning of such error; in particular, the segment size and averaging performed in order to establish the “rms error.” It is strongly advised that TWR test tolerances be developed independently of vendor supplied software, and verification of the satisfaction of TWR test tolerances be performed independently of vendor supplied software. In addition, in no case should vendor supplied software be relied upon for the specification of TWR test tolerances. However, it is vitally important that specified TWR test tolerances be correlated in some general manner with vendor supplied “rms error,” so that test interruption may be performed if large “rms error” implies specified test tolerance exceedance above a prescribed limit. If testing occurring in real time at levels exceeding the maximum test tolerance rms error limit by 10 percent, the test needs to be interrupted. Generally, it is essential that for a precise comparison (1) the reference and control time traces be band-limited to the exact SESA frequency band of interest, and (2) the reference and control time traces be maximally correlated by way of digital pre-processing (see Annex A). After such pre-processing, recommend the reference time trace be segmented into portions that might be considered stationary, short transient (or shock) and long transient. Generally, a 10 percent tapered cosine window should be applied to each of the segments such that the characteristic part of the time trace is scaled by unity, and the end points are zero. It is assumed that good signal processing practices are used to determine the basic estimates for deciding tolerance satisfaction (see Annex B). In particular, this may mean balancing the statistical random and bias error in the estimates. ASD and mean-square envelope estimates are susceptible to statistical processing errors that may distort the resulting estimates.

(1) Stationary Gaussian or non-Gaussian (may include discrete components):

(a) Frequency domain: For a cosine windowed segment represented by a Gaussian or non-Gaussian stationary random time trace, tolerances are placed upon ASD estimates. The control time trace ASD estimate is to be consistent with the tolerances given in Method 514.8.

(b) Amplitude domain comparison (STTR): When the windowed segment of the reference time trace is non-Gaussian (incorporates skewness, kurtosis or both skewness and kurtosis), recommend the plotting of the reference and control along orthogonal axes be initially performed for visual inspection. This visual inspection should then be followed by an empirical quantile plot of reference time trace amplitudes versus control time trace amplitudes (qq plot). The qq point plot should approach a straight line at forty-five degrees to each axis. Confidence intervals on this line according to the sample size can be used for tolerance specification STTR. Histogram plots of the reference and control time traces for enhanced tail structure may provide useful visual inspection, and can be used for tolerance specification for STTR. Finally, estimates of the non-Gaussian probability distribution parameters may be compared between the reference and the control time traces, exercising caution since the parameter value estimates are subject to quite restrictive statistical error considerations. For a zero mean reference time trace, ensure single estimates of the overall time trace sample variance are within ±10 percent of the reference time trace. Probability density of reference and control signals should be compared to observe skewness and kurtosis characteristics.

(2) Shock:

(a) Frequency domain: For an appropriately windowed segment represented by a shock, ensure the tolerance on the control time trace SRS estimate with 5 percent critical damping is within -6dB and+ 3dB of the reference time trace SRS estimate for at least a one-twelfth octave bandwidth resolution.

(b) Amplitude domain: For the segment, ensure the major (maximum absolute magnitude) positive and negative peaks (not to exceed 10 percent of all the reference time trace peaks in number) in the control time trace are within ± 20 percent magnitude of the corresponding peaks in the reference time trace (peak correspondence is based upon the fact that the control and reference time traces have zero phase shift between them).

(3) Nonstationary (Product Model):

(a) Amplitude domain: For an appropriately windowed segment that can be represented by the "Product Model," suggest the short-time average estimate of the control time trace envelope (time average root-mean-square level) be within ±1 dB of the short-time average estimate of the reference time trace envelope, where the short-time averaging time (and time shift in average time estimates) is not to exceed 1percent of the total duration of the reference time trace.

(b) Frequency domain comparison: Ensure the normalized ASD estimate for the control time trace is within ±3.0 dB (ratio of approximately 2) of the normalized ASD estimate for the reference time trace over a significant portion of the bandwidth. Note: this may seem a broad tolerance bound but generally the normalized ASD estimates have a restricted number of statistical degrees-of-freedom.

Annex A illustrates processing for test tolerance satisfaction. Annex B provides a table of analytical formulas and some preliminary test tolerance specifications that may be used to formally specify tailored test tolerance (in particular, for STTR). In cases where specified tolerances cannot be met, achievable tolerances should be established and agreed to by the cognizant engineering authority and the customer prior to initiation of the test.

Test interruptions can result from multiple situations. The following paragraphs discuss common causes for test interruptions, and recommended paths forward for each. Recommend test recording equipment remain active during any test interruption if the excitation equipment is in a powered state.

4.3 Test Interruption.

Test interruptions can result from a number of situations that are described in the following paragraphs.

4.3.1 Interruption Due To Laboratory Equipment Malfunction.

a. General. See Part One, paragraph 5.11, of this Standard.

b. Specific to this Method. When interruptions are due to failure of the laboratory equipment, analyze the failure to determine root cause. Drive, control and response time traces should be evaluated to ensure that no undesired transients were imparted to the test materiel during the test equipment failure. If the test item was not subjected to an over-test condition as a result of the equipment failure, repair the test equipment or move to alternate test equipment and resume testing from the point of interruption. If the test item was subjected to an over-test condition as a result of the equipment failure, notify the test engineer or program engineer responsible for the test materiel immediately. Conduct a risk assessment based on factors such as level and duration of the over-test event, spectral content of the event, cost and availability of test resources, and analysis of test specific issues to establish the path forward. In all cases, archive and analyze all available time trace information including drive, control, reference and monitor time traces, and thoroughly document the results. See Annex A for descriptions of common test types, and a general discussion of test objectives.

4.3.2 Interruption Due To Test Materiel Operation Failure.

Failure of the test materiel to operate as required during operational checks presents a situation with several possible options. Failure of subsystems often has varying degrees of importance in evaluation of the test materiel integrity. Selection of one or more options from a through c below will be test specific.

a. The preferable option is to replace the test item with a "new" one, and restart the entire test.

b. An alternative is to replace/repair the failed or non-functioning component or assembly with one that functions as intended, and restart the entire test. Conduct a risk analysis prior to proceeding since this option places an over-test condition on the entire test item, except for the replaced component. If the non-functioning component or subsystem is a line replaceable unit (LRU) whose life-cycle is less than that of the system test being conducted, it may be allowable to substitute the LRU and proceed from the point of interruption.

c. For many system level tests involving either very expensive or unique materiel, it may not be possible to acquire additional hardware for re-test based on a single subsystem failure. For such cases, perform a risk assessment by the organization responsible for the system under test to determine if replacement of the failed subsystem and resumption of the test is an acceptable option. If such approval is provided, the failed component should be re-tested at the subcomponent level.

NOTE: When evaluating failure interruptions, consider prior testing on the same test item and consequences of such. (See Part One, paragraph 5.19).

4.3.3 Interruption Due To A Scheduled Event.

There are often situations in which scheduled test interruptions will take place. For example, in a tactical transportation scenario, the payload may be re-secured to the transport vehicle periodically (i.e., tie-down straps may be re-secured at the beginning of each day). Endurance testing often represents a lifetime of exposure; therefore it is not realistic to expect the payload to go through the entire test sequence without re-securing the tie-downs as is done in a tactical deployment. Many other such interruptions, to include scheduled maintenance events, are often required over the life-cycle of materiel. Given the cumulative nature of fatigue imparted by dynamic testing, it is acceptable to have test interruptions that are correlated to realistic life-cycle events. Document all scheduled interruptions in the test plan and test report.

4.3.4 Interruption Due to Exceeding Test Tolerances.

Exceeding the test tolerances defined in paragraph 4.2.2, or a noticeable change in dynamic response may result in a manual operator-initiated test interruption or an automatic interruption when the tolerances are integrated into the control strategy. In such cases, check the test item, fixture, and instrumentation to isolate the cause. In general, the vendor means of assessing the test adequacy in real time as described in Paragraph 4.2.2c will be relied upon (based upon its general correlation to the specified test tolerances) for initiating test interruption. More detailed test tolerance assessment is completed after the test has been performed. Time average root-mean-square error between the reference and the control time traces that is above a test tolerance limit of 10 percent will be adequate for initiation of test interruption.

a. If the interruption resulted from a fixturing or instrumentation issue, correct the problem and resume the test.

b. If the interruption resulted from a structural or mechanical degradation of the test item, the problem will generally result in a test failure and requirement to re-test unless the problem is allowed to be corrected during testing. If the test item does not operate satisfactorily, see paragraph 5 for failure analysis, and follow the guidance in paragraph 4.3.2 for test item failure.

4.4 Instrumentation.

In general, acceleration will be the quantity measured to meet the specification for the selected procedure, however similar instrumentation concerns apply to other sensors. Ensure laboratory acceleration control measurements correspond to field acceleration reference measurements. This is usually accomplished by mounting the test item accelerometer for control in the same SESA location as that on the field measurement materiel from which the reference time trace was extracted.

a. Accelerometer. In the selection of any transducer, one should be familiar with all parameters provided on the associated specification sheet. The device may be of the piezoelectric or piezoresistive type. Key performance parameters for an accelerometer follow:

(1) Frequency Response: A flat frequency response within ± 5 percent across the frequency range of interest is required.

(2) Transverse sensitivity should be less than or equal to 5 percent.

(3) Nearly all transducers are affected by high and low temperatures. Understand and compensate for temperature sensitivity deviation as required. Temperature sensitivity deviations at the test temperature of interest should be no more than ±5% relative to the temperature at which the transducer sensitivity was established.

(4) Base Strain sensitivity should be evaluated in the selection of any accelerometer. Establishing limitations on base strain sensitivity is often case specific based upon the ratio of base strain to anticipated translational acceleration.

(5) Amplitude Linearity: It is desired to have amplitude linearity within 1 percent from 5 percent to 100 percent of the peak acceleration amplitude required for testing.

b. Other measurement devices. Any other measurement devices used to collect data must be demonstrated to be consistent with the requirements of the test.

c. Signal conditioning. Use only signal conditioning that is compatible with the instrumentation requirements of the test, and is compatible with the requirements and guidelines provided in paragraph 6.1, reference b.

4.5 Test Execution.

4.5.1 Preparation for Test.

Carefully examine the reference time trace for validity. Ensure the reference time trace is band limited according to the band limits of the exciter and control system software. By filtering, remove any high low-frequency components that will cause a displacement over-travel condition or velocity limit violation for the exciter. Make force requirement estimates based upon peak acceleration in the reference time trace, and the overall mass to be driven by the exciter, and compare this to the exciter force limits. If possible, integrate the acceleration time trace to obtain a velocity trace, and subsequently integrate the velocity trace to obtain a displacement trace to ensure the exciter is capable of reproducing the acceleration time trace without impacting its stops. Impacting stops, even in a cushioned hydraulic actuator, will typically result in materiel damaging accelerations. If integration is impractical or deemed likely inaccurate, the system may be operated using a dummy mass to determine if the available exciter stroke is sufficient. Generally, the vendor software estimates for maximum velocity and displacement should be verified, and some advanced signal processing procedures should be applied.

CAUTION: Integration is a difficult task that may provide unreliable answers. Using a technique such as a wavelet transformation, recommend removal of DC bias or very low frequency drift that falls below the minimum frequency of interest without imposing a phase lag.

4.5.1.1 Preliminary Steps.

Deciding upon the strategy for TWR compensation of the reference time trace, i.e., determining the exciter drive voltage, is a very important and potentially time-consuming task. The vendor approach to reference time trace compensation must be clearly understood. The advantages and disadvantages of time and frequency compensation error reduction strategies must be clearly understood. Boundary conditions and impedance mismatches almost always require maximum use of all the vendor software strategies for compensation. Use of exciter slip tables present special challenges for reference time trace compensation. Vendor software will generally allow compensation on (1) a band limited random signal, (2) a reduced level of the reference time trace, or (3) the full level reference time trace as the test progresses or as accumulated from previous testing at full level. Some vendor software may allow different compensation functions (transfer functions) on different portions of the reference time trace. It is recommended that testing be initially performed on a dynamic simulant item that represents the dynamic properties of the materiel to be tested to ensure the reference time trace can be properly compensated and accurately replicated. Remember that the bandwidth of the control time trace reflects the response of the dynamic simulation item or the materiel, and may be substantially broader than the bandwidth of the reference time trace. TWR “control” is generally active only over the bandwidth of the reference time trace, allowing uncompensated response outside of this bandwidth. Vendor software may permit control beyond the band limit of the reference time trace. If the bandwidth differences (reference versus control) can be detected early on, this will be helpful in interpreting the results of the test, particularly with respect to meeting test tolerances.

4.5.1.2 Pretest Checkout.

Verify that each of the following check list items is established prior to initiation of the test

a. Test fixture requirements.

b. Test fixture modal survey requirements / procedure.

c. Test item/fixture modal survey requirements / procedure.

d. Control and monitor measurement locations correlate with the configuration for which the reference time trace was obtained.

e. Test tolerances.

f. Requirements for combined environments.

g. Test schedule(s) and duration of exposure(s).

h. Axes of exposure.

i. Test shutdown procedures for test equipment or test item problems, failures, etc.

j. Test interruption recovery procedure. (See paragraph 4.3.)

k. Test completion criteria including any post processing for a refined tolerance assessment (STTR).

l. Test requirements (force, acceleration, velocity, displacement) can be met. Seek approval for variation if required. Document any variation.

m. Allowable adjustments to test item and fixture (if any); these must be documented in test plan and the test report.

n. Adequate digital data storage requirements.

4.5.2 Procedure Specific.

The following steps provide the basis for collecting the necessary information under TWR testing.

4.5.2.1 Procedure I - SESA Replication of a Field Measured Materiel Time Trace Input/Response.

Step 1 Following the guidance of paragraph 6.1, reference b, select the test conditions and mount the test item (or dynamic simulant item) on the vibration exciter. Select accelerometers and analysis techniques that meet the criteria outlined in paragraph 6.1, reference b.

Step 2 If required; perform an operational check on the test item at standard ambient conditions. If the test item operates satisfactorily, proceed to Step 3. If not, resolve the problems and repeat this step.

Step 3 Subject the test item (or dynamic simulant) to the system identification process that determines the compensated exciter drive voltage. This may include a careful look at the component parts of the reference time trace, i.e., stationary vibration, shock, transient vibration; and determination of the potential time variant properties of the compensating function. If a dynamic simulant is used, then replace the dynamic simulant with the test item after compensation.

Step 4 Subject the test item in its operational configuration to the compensated waveform. It is often desirable to make an initial run at less than full level to ensure proper dynamic response and validate instrumentation functionality.

Step 5 Record necessary data, paying particular attention to the vendor software supplied test error indicator and, in general, the control acceleration time trace that can be post processed to demonstrate tolerance satisfaction.

Step 6 Perform an operational check on the test item and record the performance data as required. If failure is noted, follow the guidance in paragraph 4.3.2.

Step 7 Repeat Steps 4, 5, and 6 for the number of replications called out in the requirements document, or a minimum of three times for statistical confidence provided the integrity of the test configuration is preserved during the test.

Step 8 Document the test series including the saving of all control and monitor digital time traces, and see paragraph 5 for analysis of results.

4.5.2.2 Procedure II - SESA Replication of an Analytically Specified Materiel Time Trace Input/Response.

Follow the guidance provided in Steps 1-8 in Paragraph 4.5.2.1 subsequent to scaling the reference time trace per the scaling guidance provided in paragraph 1.2.6.

4.5.3 Data Analysis.

Ideally, information from the control time trace in the time and frequency domains should be nearly identical to that information contained in the reference time trace. Vendor supplied test error assessment provides a preliminary indication of the replication efficacy. If vendor supplied test error assessment consistently displays less than, e.g., 5 percent time average rms error over blocks of reference/control data, additional analysis may be unnecessary. For production testing, reliance on consistency of vendor supplied rms error is highly desirable. For single item tests that are unique and for which vendor rms error provides values greater than acceptable, then differences between the reference and control time traces must be assessed in detail. The following guidance is provided.

a. Rudimentary analysis to ensure the test tolerances are met is usually performed within the TWR vendor software. Laboratory personnel should consult the vendor supplied TWR system documentation, and clearly understand the determination of these test tolerances. In most cases, this will require direct contact with the vendor of the TWR system.

b. More extensive data analysis can be performed to ensure test tolerances are met based upon reference and control time trace ASCII files, with off line specialized software according to procedures illustrated in Annex A and discussed in Annex B.

c. Detailed data analysis for purposes of establishing parameters for a random process or other purposes may be performed, but must be consistent with the information provided in the Annexes, and best data processing procedures as defined in paragraph 6.1, references a or b. Such detailed analysis may be beyond the scope of defined tolerances, and is to be used for report information purposes only.

d. Processing of monitor time trace information for modeling, failure assessment, or other purposes must follow the same guidelines as for the control time trace.

5. ANALYSIS OF RESULTS.

In addition to the guidance provided in Part One, paragraphs 5.14 and 5.17; and Part One, Annex A, Tasks 405 and 406, the following information is provided to assist in the evaluation of the test results. Analyze in detail any failure of a test item to meet the requirements of the specification, and consider related information such as:

a. Information from the control accelerometer configuration, including a digital record of the control time trace.

b. The vendor TWR software test tolerance information.

c. Application of one or more of the techniques illustrated in Annex A and elaborated on in Annex B for detailed comparison of the reference time trace to the control time trace.

5.1 Physics of Failure.

Analyses of vibration related failures must relate the failure mechanism to the dynamics of the failed item and to the dynamic environment. It is insufficient to determine that something failed due to high cycle fatigue or wear. Include in failure analyses a determination of resonant mode shapes, frequencies, damping values and dynamic strain distributions, in addition to the usual material properties, crack initiation locations, etc.

5.2 Qualification Tests.

When a test is intended to show formal compliance with contract requirements, recommend the following definitions:

a. Failure definition. Materiel is deemed to have failed if it suffers permanent deformation or fracture; if any fixed part or assembly loosens; if any moving or movable part of an assembly becomes free or sluggish in operation; if any movable part or control shifts in setting, position or adjustment, and if test item performance does not meet specification requirements while exposed to operational or endurance test levels. Ensure this statement is accompanied by references to appropriate specifications, drawings, and inspection methods.

b. Test completion. A TWR qualification test is complete when all elements of the test item have successfully passed a complete test. When a failure occurs, stop the test, analyze the failure and repair the test item. Continue the test until all fixes have been exposed to a complete test. Qualified elements that fail during extended tests (tests extended beyond LCEP requirements) are not considered failures, and can be repaired to allow test completion.

5.3 Other Tests.

For tests other than qualification tests, prepare success and/or failure criteria and test completion criteria that reflect the purpose of the tests.

6. REFERENCE/RELATED DOCUMENTS.

6.1 Referenced Documents.

a. Bendat, Julius S. and Allan G. Piersol, Random Data Analysis and Measurement Procedures, 4th Edition, John Wiley & Sons, Inc., New York, 2010.

b. Handbook for Dynamic Data Acquisition and Analysis, IEST-RD-DTE 012.2, Institute of Environmental Sciences and Technology, Arlington Place One, 2340 S. Arlington Heights Road, Suite 100, Arlington Heights, IL 60005-4516.

c. Merritt, Ronald G., "Application of Mixed Effects Models to a Collection of Time Trace Product Models," Proceedings of the 77th Shock and Vibration Symposium, Nov. 2006; Shock & Vibration Exchange (SAVE), 1104 Arvon Road, Arvonia, VA 23004.

d. Stuart, Alan and J. Keith Ord, Kendall's Advanced Theory of Statistics, 5th Edition Volume 1 Distribution Theory, Oxford University Press, New York NY, 1987.

6.2 Related Documents.

a. Bickle, Larry W. and Ned R. Keltner, Estimation of Transient Measurement Errors, SAND78-0497, August 1978.

b. Shock and Vibration Handbook, 5th Edition, Edited by Cyril M. Harris and Allan G. Piersol, McGraw-Hill, New York NY, 2002.

c. Egbert, Herbert W. "The History and Rationale of MIL-STD-810 (Edition 2)", January 2010; Institute of Environmental Sciences and Technology, Arlington Place One, 2340 S. Arlington Heights Road, Suite 100, Arlington Heights, IL 60005-4516.

(Copies of Department of Defense Specifications, Standards, and Handbooks, and International Standardization Agreements are available online at https://assist.dla.mil.

Requests for other defense-related technical publications may be directed to the Defense Technical Information Center (DTIC), ATTN: DTIC-BR, Suite 0944, 8725 John J. Kingman Road, Fort Belvoir VA 22060-6218, 1-800-225-3842 (Assistance--selection 3, option 2), http://www.dtic.mil/dtic/; and the National Technical Information Service (NTIS), Springfield VA 22161, 1-800-553-NTIS (6847), http://www.ntis.gov/.

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METHOD 525.2, ANNEX A
SESA POST-TEST ANALYSIS ILLUSTRATION FOR TEST TOLERANCE ASSESSMENT

1. PURPOSE.

This Annex is designed to provide general guidelines for post-test analysis for SESA TWR testing. It displays some potentially useful tools for comparison of "reference" and "control" time traces and processing the difference between these time traces. Post-test analysis provides insight into development of test tolerance limits for single axis TWR.

2. GENERAL PHILOSOPHY FOR TWR TESTING.

Broadband TWR, i.e., from 5 Hz to 2000+ Hz, is relatively new to dynamic laboratory testing with electrodynamic force exciters. The same comment applies to electrohydraulic force exciters only over a more limited bandwidth. The philosophy for TWR testing, including test tolerance development, is still evolving. The post-test analysis rationale displayed below will doubtlessly be augmented/refined/enhanced with portions eliminated, however fundamentals behind the analysis rationale will remain.

The general term *"replication error"* will be used with regard to the comparison of the difference between the control and reference time traces. SESA post-test analysis quantitatively compares the deterministic test input reference time trace, $r(t)\left(\textit{or sampled sequence } r[n] \textit{ for } n=1,2,\ldots,N\right), \textit{ symbolic } "r,"$ with the stochastic test output control time trace, $c(t)\left(\textit{or sampled sequence } c[n] \textit{ for } n=1,2,\ldots,N\right), \textit{ symbolic } "c."$ For comparison, it is convenient to have available a stochastic difference time trace defined as:

$$s(t)=c(t)-r(t)\left(\textit{or sampled sequence } \; s[n]=c[n]-r[n], n=1,2,3,\ldots,N\right), \textit{ symbolic } "s."$$

The difference time trace represents the "replication error." The reference and control time traces are assumed to be perfectly correlated in time so that the difference time trace is valid, and generally vendor software is very reliable in supplying reference and control digital time traces that are perfectly correlated. A **time/amplitude point-by-time/amplitude point** (TPP) assessment of the time traces can be made, and an estimate of replication error determined. Annex B addresses in more detail the statistical implications of TPP. Generally, vendors will make available a drive voltage time trace for potential use in understanding the test limitations, i.e., fixture resonance compensation, impedance mismatch, etc. This time trace must be preprocessed in the same manner as $r, c,$ and s. The drive time trace is of no concern in the illustration to follow. Discussion appears in both this Annex and Annex B concerning **time/amplitude average-by-time/amplitude average** (STA) assessment for tolerance limit analysis – an alternative to TPP. Application of these procedures for tolerance error assessment will be mentioned in this Annex and in Annex B. Generally, direct comparison of time average estimates of *r* and *c* is much less desirable than either examining statistics on or statistics on a time averaged version of *s*. Interpretation of differences between time average estimates is more difficult.

3. DESCRIPTION OF REFERENCE TIME TRACE.

The time trace selected for illustration is one unidentified band limited field measured acceleration time trace used to assess the performance of the vendor software for a single axis exciter configuration. Test item configuration including fixturing was of no concern. The simplicity of the TWR test provides for replication error that is smaller than that encountered in general testing scenarios where boundary conditions and impedance mismatches become important. Figure 525.2A-1 displays the unprocessed reference time trace acceleration measured in the field.

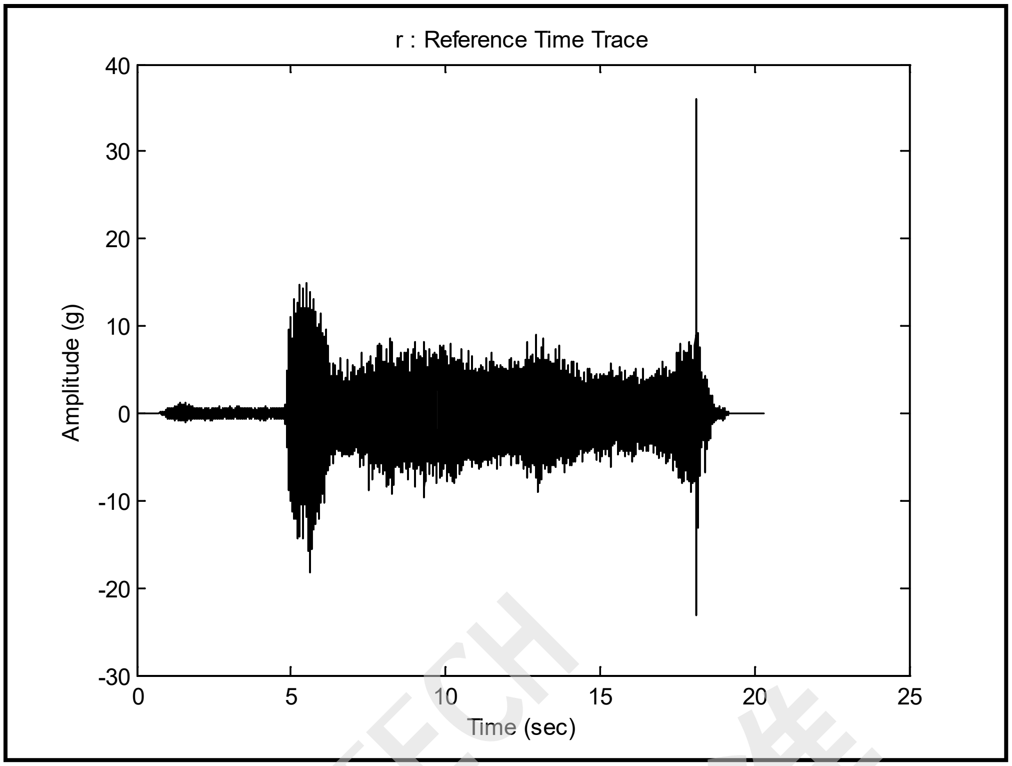


Figure 525.2A-1. Field measured acceleration reference time trace.

The time trace is band limited between 1 Hz and 2000 Hz, and consists of an initial and final low level stationary random vibration (augmented with some analytically generated zeros), along with a form of comparatively high level transient vibration, stationary random vibration and shock in succession. This visual assessment of the reference time trace is a key to examining the test performance adequacy. Under standard vendor vibration and shock system software, it would not be possible to test materiel to this form of time trace. The time trace was submitted for TWR testing under ambient conditions on an electrodynamic exciter using a vendor-supplied TWR software package. The "control accelerometer" was mounted on both the exciter head and on a conventional slip table. Even though TWR "control" is between 10 Hz and 2000 Hz, the sample rate of the reference time trace ASCII file is 25600 samples per second. The particular TWR vendor software re-sampled the waveforms to 24576 samples per second prior to testing. The Nyquist frequency is 24576/2=12288 Hz. Most frequency domain plots will be restricted to 4000 Hz, and basic TWR control is out to 2000 Hz. The field measured time trace should display a bandwidth that exceeds the TWR control bandwidth to as much as an octave above and below the upper and lower control bandwidth limits, respectively. For demonstration of the effect of different boundary conditions, results of the testing will be displayed for the control time trace from the exciter head (designated (H)) and the exciter slip table (designated (S)).

4. TIME TRACE PRE-PROCESSING.

4.1 Introduction.

Not many post-test analysis procedures (independent of vendor supplied test analysis) have been formally established and agreed upon for quantifying the replication error. For one-of-a-kind type testing with a unique reference time trace, some reliance should be made upon custom software in post-test analysis to verify test tolerance satisfaction.

Figure 525.2A-2 displays the TWR control time traces for (H) and (S) configurations (along with the same reference time trace) prior to beginning of preprocessing where the time traces have been truncated for convenience.

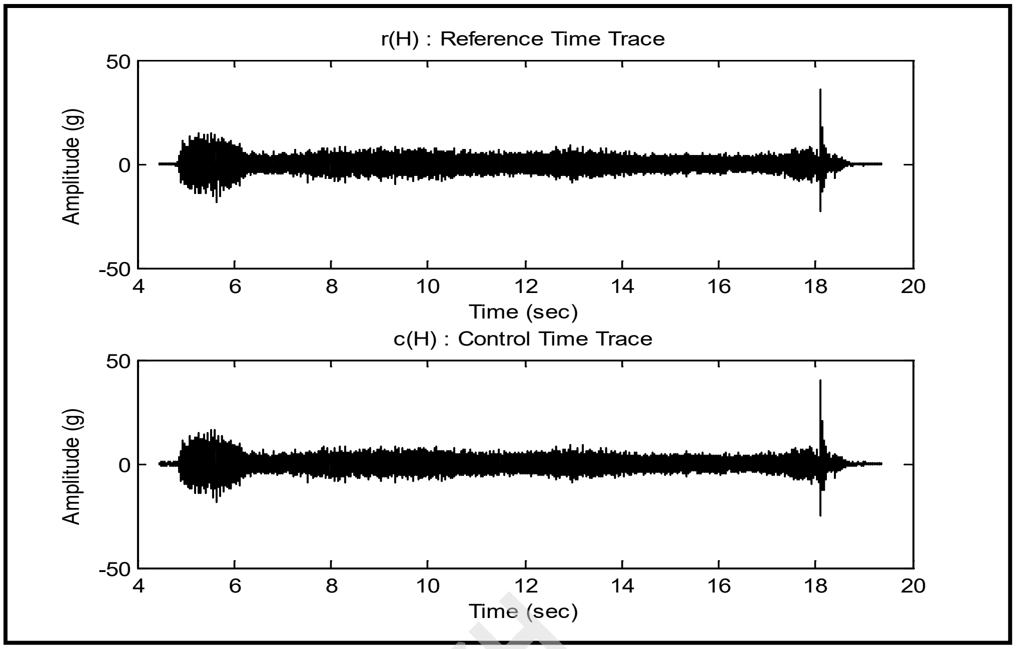


Figure 525.2A-2a. Exciter head (H) (reference/control time traces prior to post-test preprocessing).

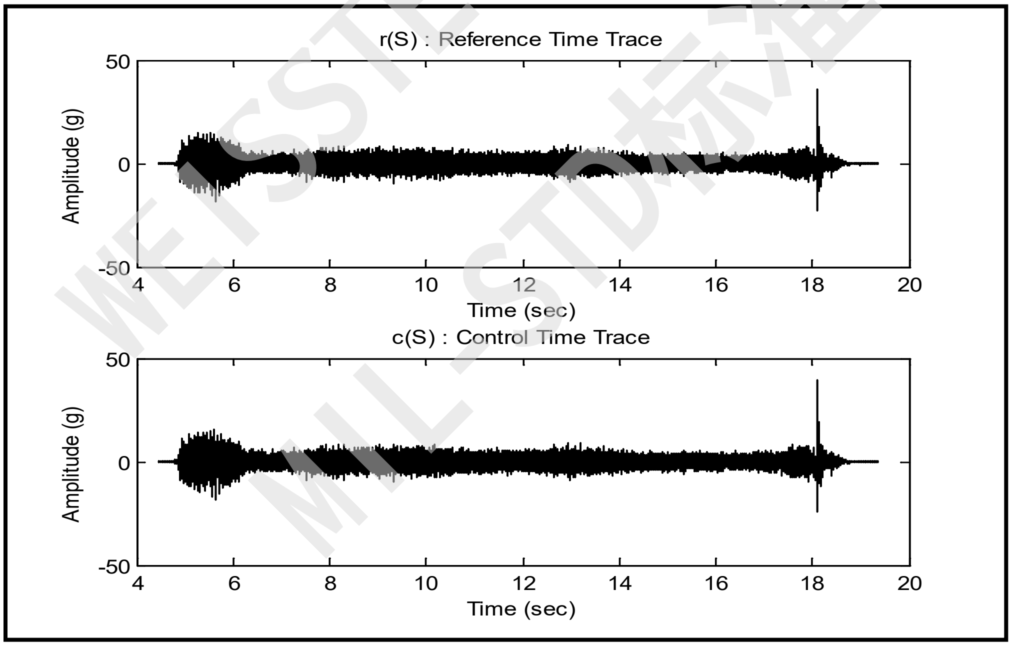


Figure 525.2A-2b. Exciter slip table (S) (reference/control time traces prior to post-test preprocessing).

Before the reference and control time traces are processed and the difference time trace is generated, some preprocessing is necessary. Preprocessing must be performed in both the time and frequency domains. The following preprocessing procedures will be discussed in turn:

a. Frequency Band Limiting.

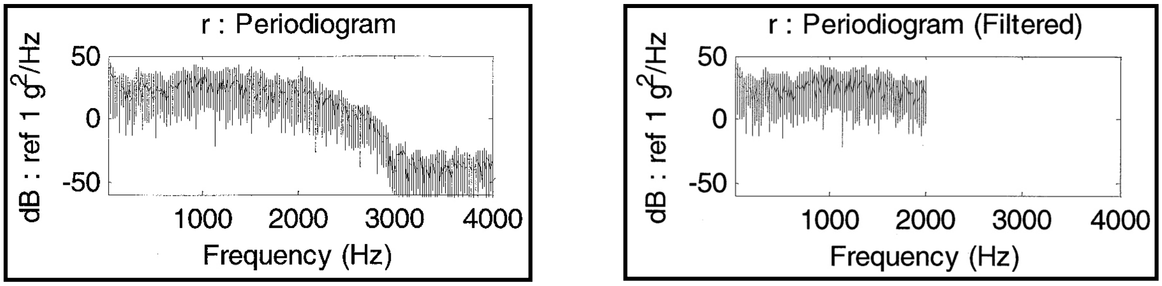
b. Time Trace Correlation.

c. Time Trace Segment Identification.

4.2 Frequency Band Limiting.

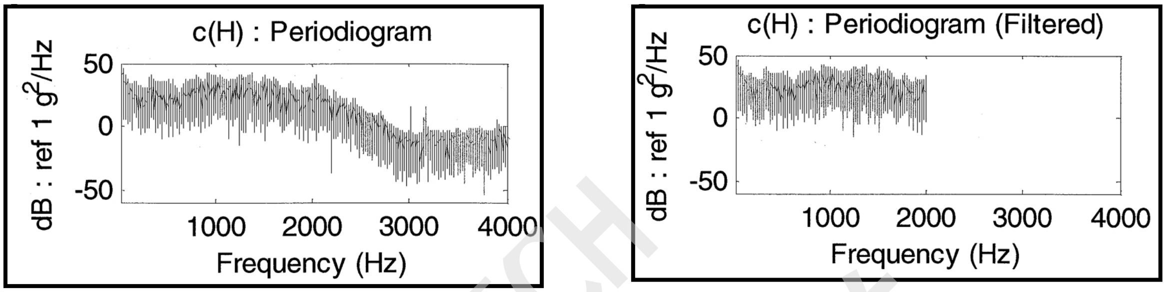
The objective of frequency band limiting is to ensure for time trace comparison, the reference and control time traces exist over the same exact frequency band (generally a bandwidth coincident with the TWR control bandwidth). The importance of this operation cannot be over emphasized. If the control time trace has significant high frequency information not contained in the reference time trace (as a result of boundary conditions or impedance mismatch), this will be reflected in any TPP amplitude comparisons. The band pass filter to provide a common bandwidth for the time traces is selected such that the minimum of the reference bandwidth and the control bandwidths is established. This common bandwidth may be specified as, e.g., 10 Hz to 2000 Hz, or determined by examining the magnitude of a periodogram estimate for both time traces. The frequency band limiting operation is performed on both the reference and control time traces, and always performed before time trace correlation considerations. Unless the time traces are excessive in length, a single block rectangular window FFT magnitude (periodogram) plotted in dB for both the reference and control time traces is satisfactory for identifying the common bandwidth. For excessively long time traces, the Welch method of spectrum computation may be employed for common bandwidth identification. To obtain the common bandwidth, a standard bandpass filter may be applied, making sure to preserve filter phase linearity, in obtaining the reference and control time traces. Figure 525.2A-3 provides single block periodograms for the reference and control time traces before and after bandpass filtering.

NOTE: With regard to frequency band-limiting, it is very important that for any field time trace measurement program designed to provide input to TWR laboratory testing, the bandwidth of the field measurements exceeds by definition, the bandwidth of interest for laboratory testing (TWR test control bandwidth). For example, if test specifications call for a 10 Hz to 2000 Hz laboratory test bandwidth, the field time trace measurements must exceed 2000 Hz, e.g., 4000 Hz, in order to provide a reference time trace with sufficient bandwidth to compare with the unprocessed control time trace resulting from TWR laboratory testing. Less critically field measurements would have frequency content below 10 Hz, e.g., 5 Hz. The rationale behind this is as follows. Almost certainly the laboratory test will exhibit energy out of the test specification frequency band of interest or the exciter test control bandwidth as a result of mismatch of materiel/test fixture/exciter impedance/boundary conditions. To directly compare the field reference time trace (before bandwidth limiting as a TWR input) with the unprocessed laboratory control time trace, (even though the reference time trace may have been bandlimited for laboratory test), the field measured reference time trace must have a bandwidth consistent with the unprocessed laboratory control time trace, i.e., a bandwidth that encompasses the bandwidth of the unprocessed laboratory control time trace. Thus, bandlimiting for comparison of reference and control time traces must be in accord with the most significant energy in the unprocessed laboratory control time trace (that likely exceeds the test specification bandwidth). Comparison for purposes of time trace peak modeling for the reference and control time trace is particularly sensitive to frequency bandlimiting considerations. To compare reference and control time trace information in terms of the full bandwidth that the materiel experienced in laboratory test, the laboratory test control bandwidth must determine the bandwidth for comparison. In the example provided here the field measured reference time trace was bandlimited to 2000 Hz (by measurement system design without TWR consideration) thus, by necessity, in comparison, the measured reference time trace somewhat “incorrectly” controls bandwidth for comparison. As noted, TWR testing has important implications for field measurement system design.



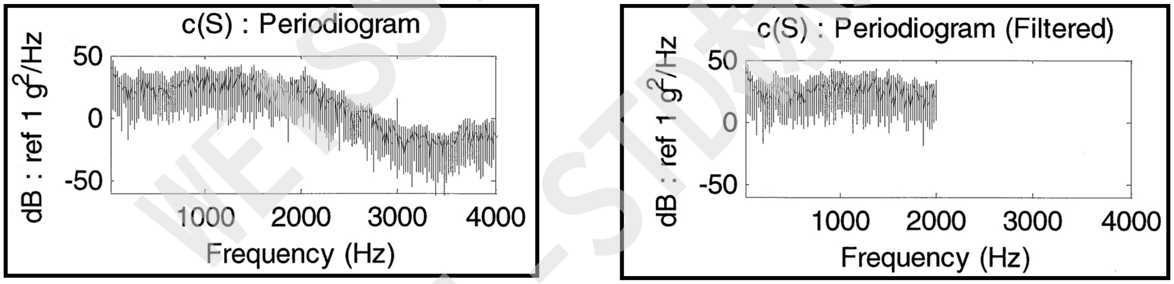
(a) Reference Time Trace

(b) Bandlimited Reference Time Trace (10 Hz – 2000 Hz)



(c) Control Time Trace Exciter Head (H)

(d) Bandlimited Control Time Trace Exciter Head (H) (10 Hz – 2000 Hz)



(e) Control Time Trace Slip Table (S)

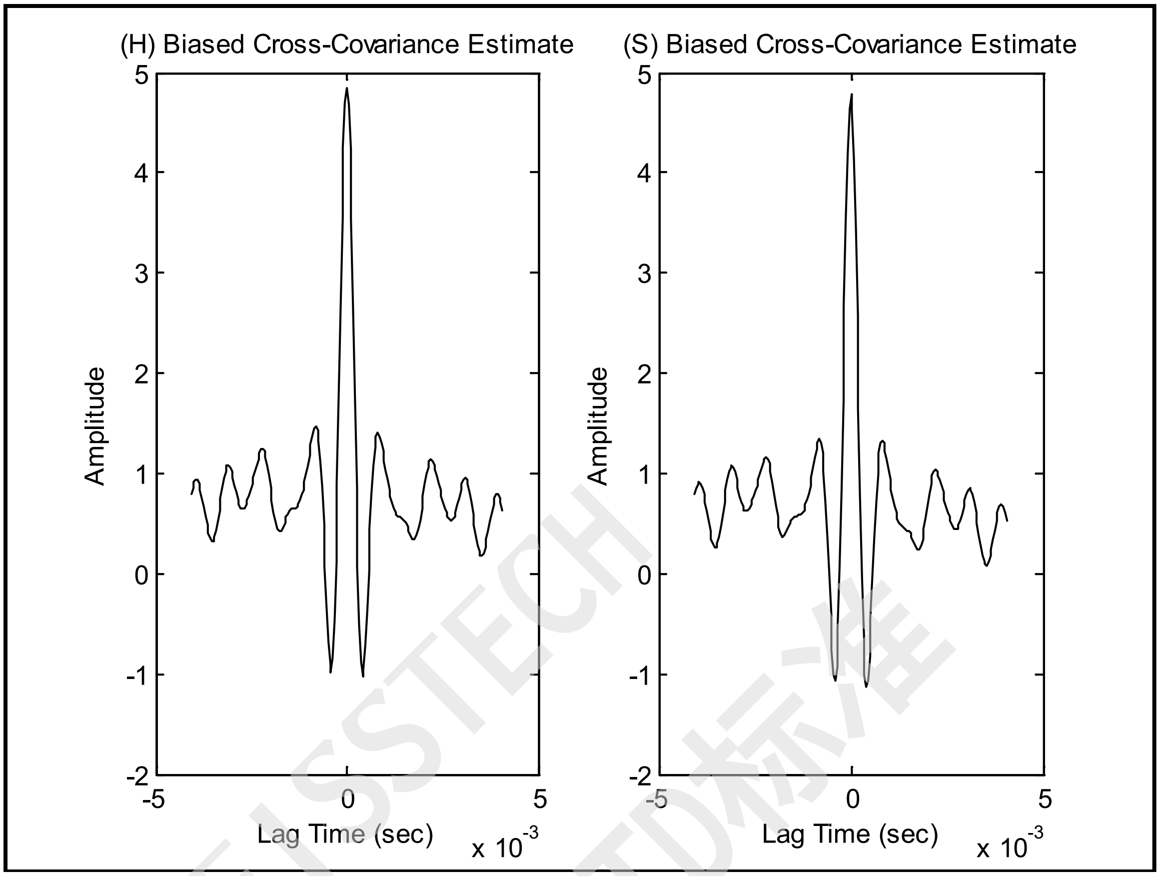
(f) Bandlimited Control Time Trace Slip Table (S) (10 Hz – 2000 Hz)

Figure 525.2A-3. Reference/control time trace periodograms for frequency band limiting through FFT window filtering.

Based upon examination of the periodograms for both time traces in Figure 525.2A-2, the very low frequency information (below 10 Hz), and the very high frequency information (above 2000 Hz) is filtered out. The frequency *analysis bandwidth* for this operation is 0.067 Hz.

4.3 Time Trace Correlation.

After a common frequency bandwidth has been established, it is essential that the band limited reference and control time traces be “perfectly” or “maximally” correlated in time (i.e., one time trace is not shifted in time relative to the other time trace) for TPP assessment. If the vendor software does not guarantee this perfect correlation in time, the degree of correlation must be checked. To perform this check and take corrective action, the cross-covariance function estimate is determined, and the time traces shifted relative to one another, such that the peak in the cross-covariance function estimate appears at the zero cross-covariance lag. This computation should be performed, if possible, on a reasonably stationary segment of the time trace. It is unnecessary to perform the correlation computations over the entire trace, but only necessary to get a long-enough segment such that the degree of shift can be determined with confidence (dependent upon the accuracy of the covariance function estimate). Figure 525.2A-4 provides a biased cross-covariance function estimate between the band-limited reference and control time traces.



(a) Bandlimited Reference and Control Time Trace (H)

(b) Bandlimited Reference and Control Time Trace (S)

Figure 525.2A-4. Cross-covariance function estimates between reference and control time traces.

By examining the cross-correlation estimate region near a lag of zero seconds, it is apparent that the reference and control time traces are in phase, and no shifting of one time trace relative to the other is necessary.

4.4 Time Trace Segment Identification.

It is tacitly assumed that the reference and control time traces are preserved in such a way that (1) they are band-limited to the exact frequency band, and (2) they are simultaneously sampled at the SESA sample rate and over the exact time interval, providing no phase shift between the traces. Conditions in (1) and (2) have been met in paragraphs 4.2 and 4.3 (in this Annex), respectively. The purpose of time trace segment identification is to break the time trace into component parts that may be assessed independently for test replication error. There is no known single analysis procedure that can ***consistently*** assess the replication error for all six forms of time trace components identified in paragraph 1.2.3 of this Method. Figure 525.2A-5 reveals the five segments into which the $r, c,$ and s time traces can be divided.

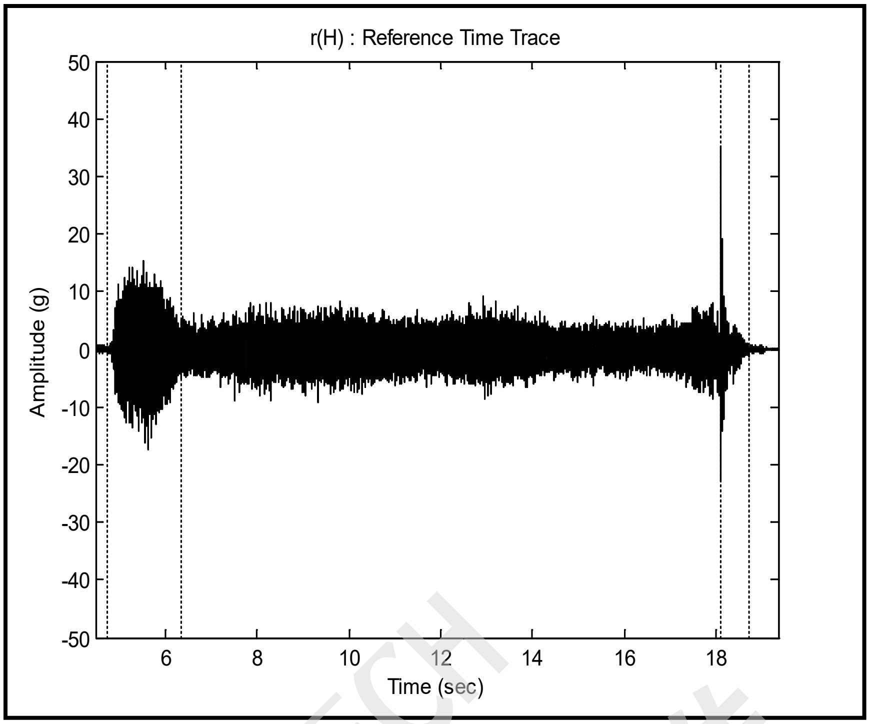


Figure 525.2A-5. Time trace segment identification from previously truncated reference time traces.

The first and fifth segments represent low level pre- and post-test acceleration of no interest for test tolerance consideration. The second segment represents a transient vibration, the third segment stationary random vibration, and the fourth segment a shock. For further processing purposes, the three segments of interest are extracted by use of a rectangular window over the duration of the segment. The three segments are displayed in Figures 525.2A-6 through 525.2A-8.

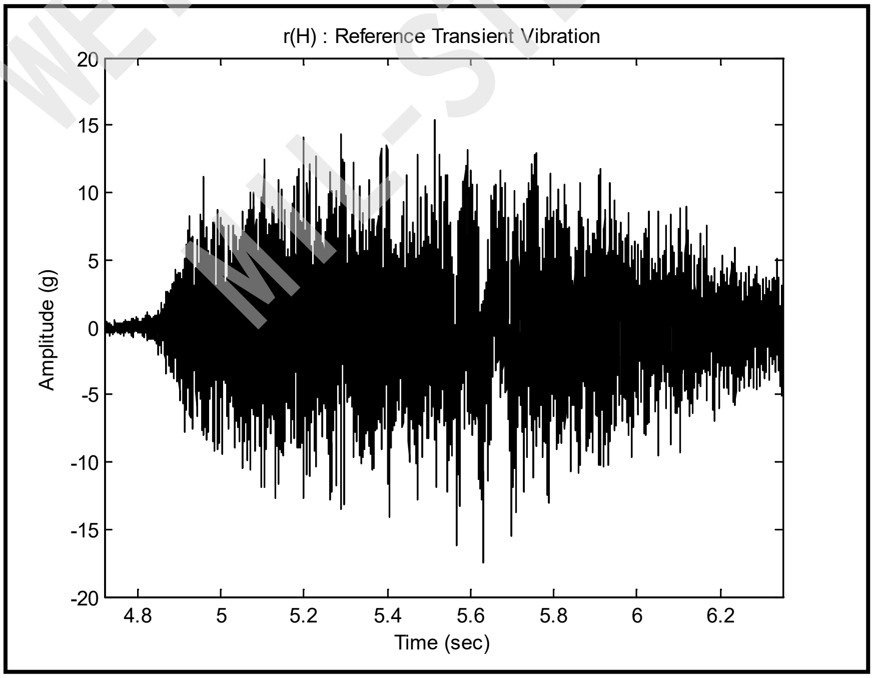


Figure 525.2A-6. Transient vibration reference time trace segment.

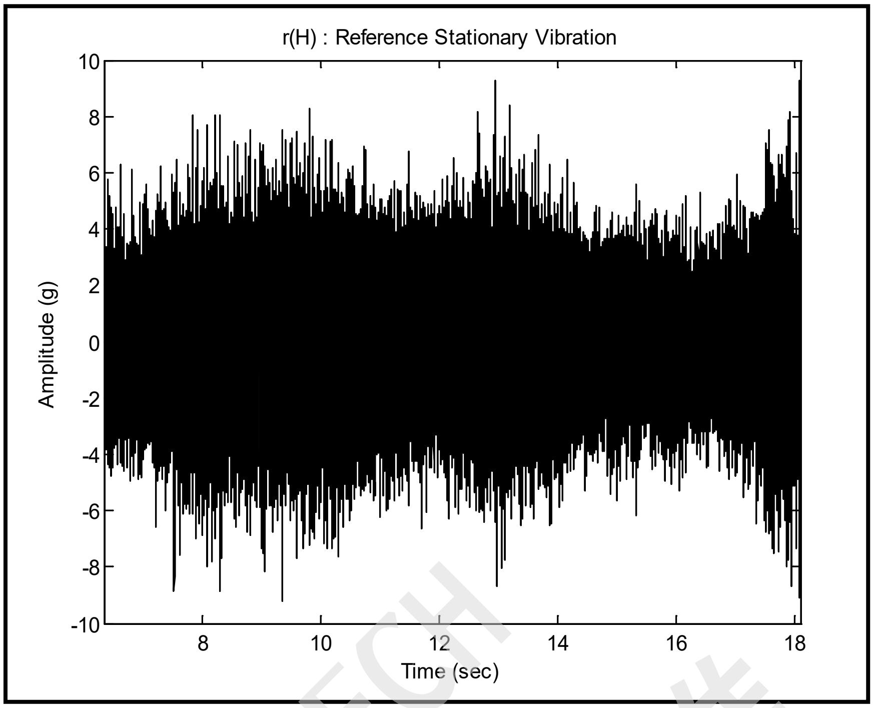


Figure 525.2A-7. Stationary random vibration reference time trace segment.

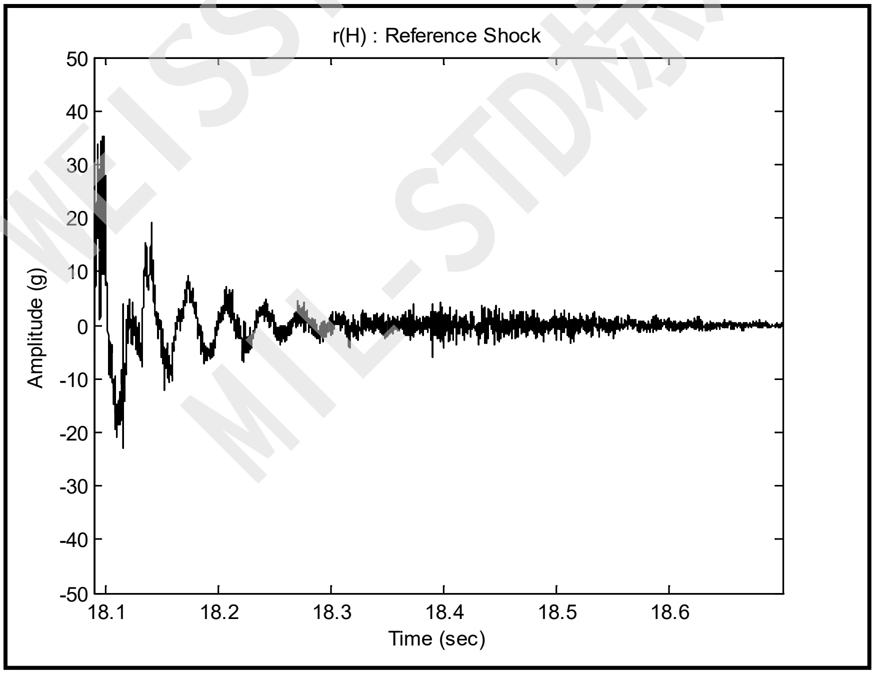


Figure 525.2A-8. Shock reference time trace segment.

For materiel particularly sensitive to a band or bands of frequencies, both time traces may be filtered (phase linearity preserved) into a number of bands, and post-processing performed on the band or bands individually. It is quite acceptable to decide and agree upon (before laboratory testing) a band-pass filter strategy that will be acceptable for assessing replication error. This form replication error assessment will not be pursued further here.

5. POST-TEST PROCESSING FOR TPP.

From pre-processing, three individual segments of different form exist along with the overall time trace. For reference purposes, the overall difference time trace along with TPP root-mean-square level are displayed in Figures 525.2A-9a and 525.2A-9b. In addition, the difference of the differences is provided in Figure 525.2A-9c.

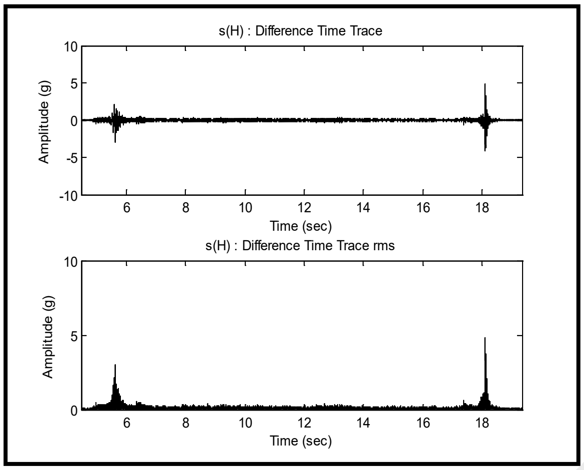


Figure 525.2A-9a. Difference Exciter (H).

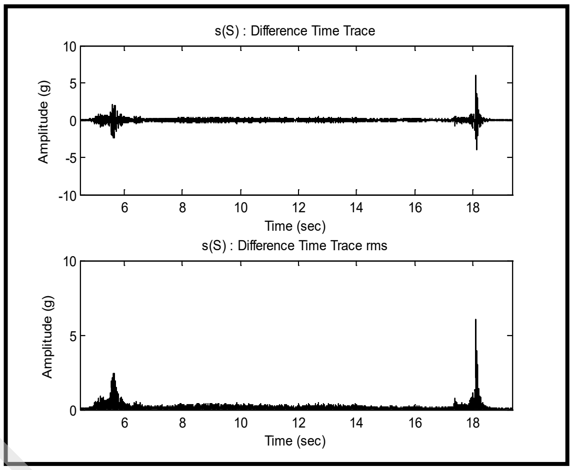


Figure 525.2A-9b. Difference Exciter (S).

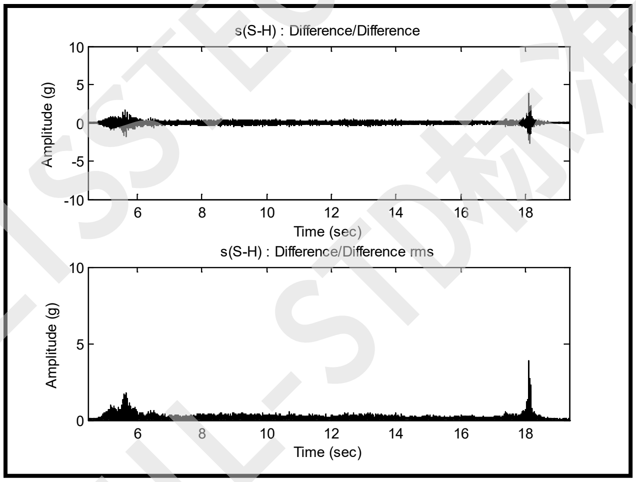


Figure 525.2A-9c. Time Trace of Difference of the Differences ((S) – (H)).

Figures 525.2A-9a-9c. Plots of overall difference time trace with root-mean-square.

In this particular case, TPP difference s(H) and s(S) may approach 5g, whereby the reference time trace was bounded by 40g in the positive and negative directions. This would suggest that, in certain parts of the time trace, the normalized random error might approach 0.125, i.e., 12.5 percent. The rudimentary overall maximum and minimum statistics for the time traces are as follows: r(H) min/max –22.84/35.24; c(H) min/max –24.28/39.76; and s(H) min/max –4.11/4.78; c(S) min/max –23.85/39.03; and s(S) min/max –3.95/6.08. The differences between response on the head of the shaker (H) and the shaker slip table (S) are reasonably nominal, so that only results for the shaker head will be provided below. When reviewing several test measurements, it is usually desirable to provide comprehensive post-test analysis on one set of measurements, and infer that similar analysis on the other measurements. The segments will now be processed in turn according to meaningful and easy to interpret estimates.

6. TPP TRANSIENT VIBRATION.

Figure 525.2A-10 displays the transient vibration time trace information, from which the general form of the transient vibration is preserved, and the difference is reasonably nominal. There is an apparent low frequency component in the time traces between 5.58 and 5.70 seconds. Such a dominant low frequency component could preclude strict product model assumptions for post processing. However, generally, the product model is reasonably robust with regard to change of frequency, i.e., the momentary change in frequency character is averaged in over the entire record length.

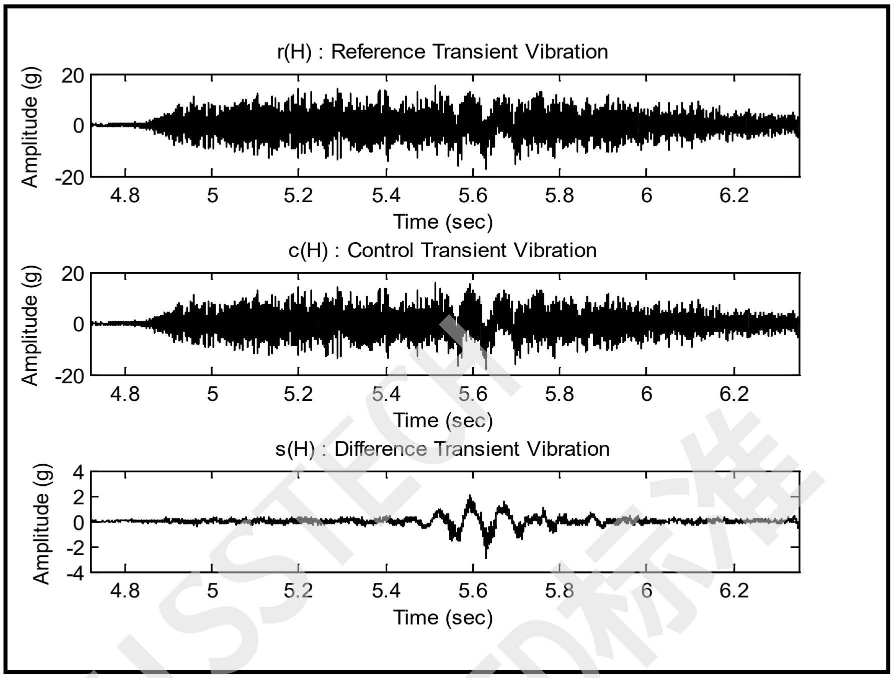


Figure 525.2A-10. Transient vibration time traces - r, c, and s.

The rudimentary overall maximum and minimum statistics for the transient vibration time trace are as follows: r min/max –17.50/15.41; c(H) min/max –18.12/16.11; and s(H) min/max –2.99/2.12.

The replication error is assessed under the product model assumption as follows:

a. Plot for r versus c (cross-plot) is generated to measure strength of TPP correlation (particularly for peaks and valleys at extremes of the cross-plot).

b. qq-plot for s is generated to examine the difference time trace for normality.

c. Root-mean-square envelopes are generated at 0.1 second averaging time for r and c under a product model assumption.

d. Normalized ASD estimates are determined for r and c under a product model assumption.

Figure 525.2A-11 plots the amplitude of r versus the amplitude c. Each individual point in the plot represents a point in time with r amplitude along the horizontal axis, and c amplitude along the vertical axis. The spread along the minor axis of this ellipsoidal form implies the difference in r and c at several time increments. In this particular case, the negative peak spread near -18g is nominal, whereas the positive peak spread near 14g demonstrates up to a 2g difference at given time increments. The spread near $r \approx c \approx 0$ is of little concern since the signal-to-noise ratio is small, and statistically independent Gaussian noise samples are being scatter plotted.

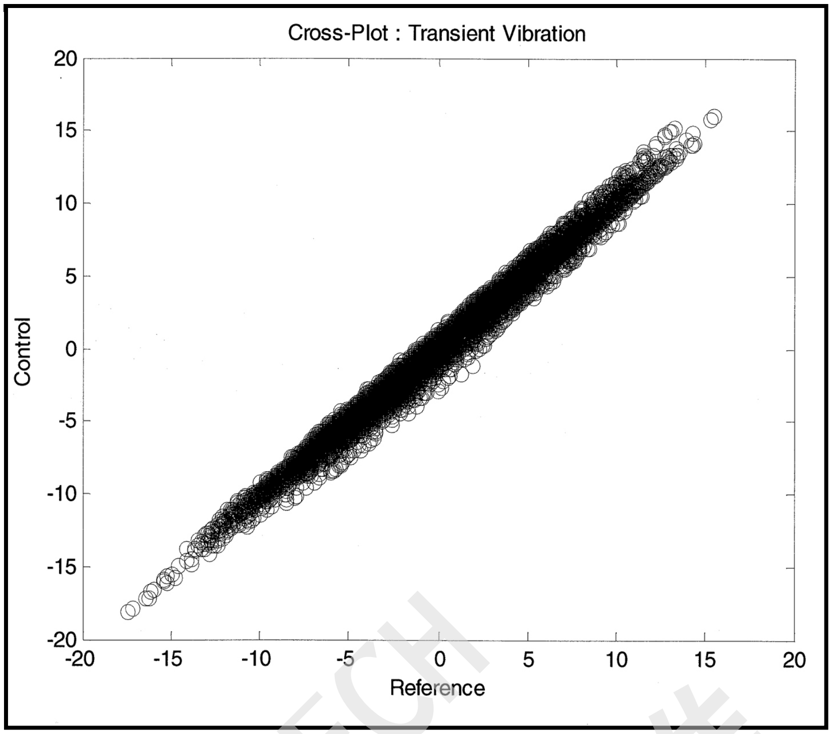


Figure 525.2A-11. r versus c cross-plot.

Figure 525.2A-12 displays the quantiles of s versus the Gaussian distribution. This figure clearly reveals that the difference between r and c is non-Gaussian, and this complicates the replication error assessment. In particular, "s" has tails that are longer that those that might be expected for a Gaussian distribution with a mean and standard deviation estimated from s.

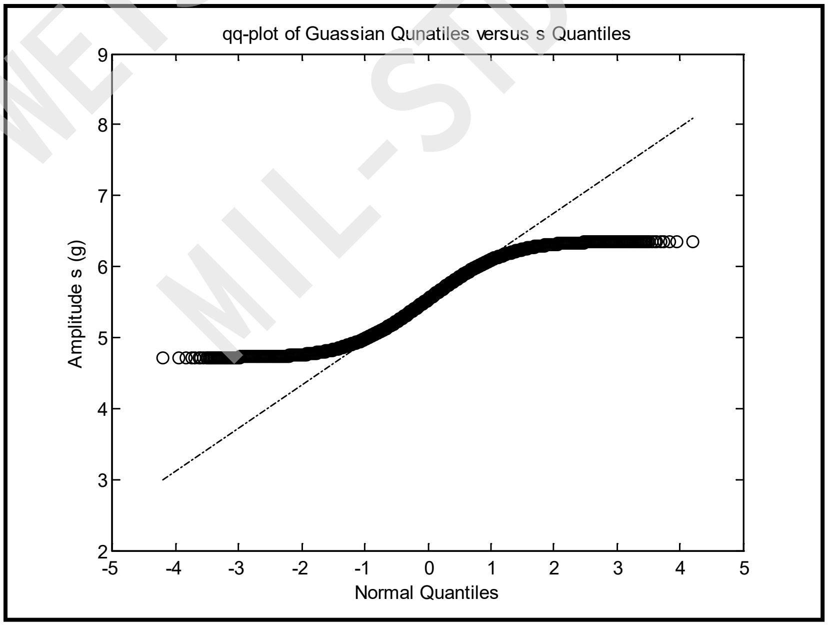


Figure 525.2A-12. Transient vibration q-q plot for s versus Gaussian.

Figure 525.2A-13 provides an overlay of envelopes of r and c in terms of root-mean-square g's for a short-time averaging increment of 0.1 seconds (STA assessment). If the product model can be assumed, the differences in root-mean-square envelope levels are a maximum of 2 percent.

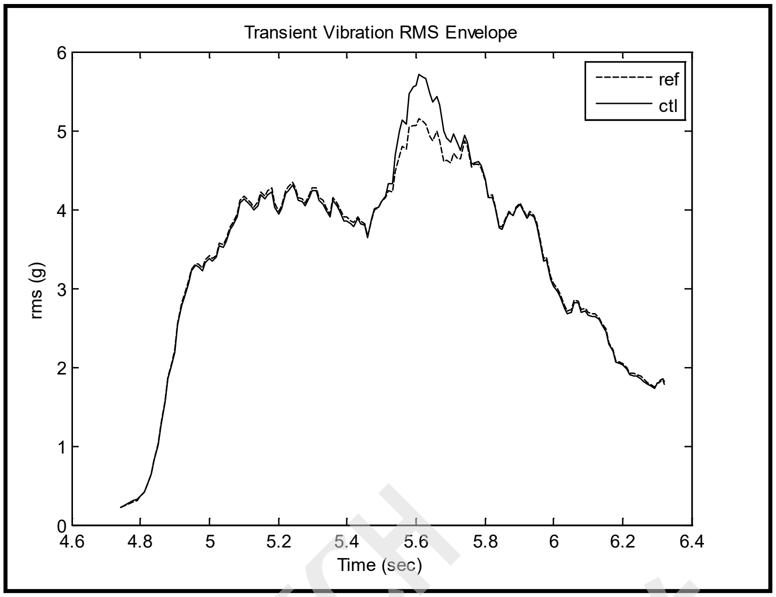


Figure 525.2A-13. Composite root-mean-square envelope estimates for r and c.

Figure 525.2A-14 provides a composite of normalized ASD estimates for r and c. The estimates were determined by one-sixth octave band frequency averaging. The normalized ASD estimates differ by less than 2 dB.

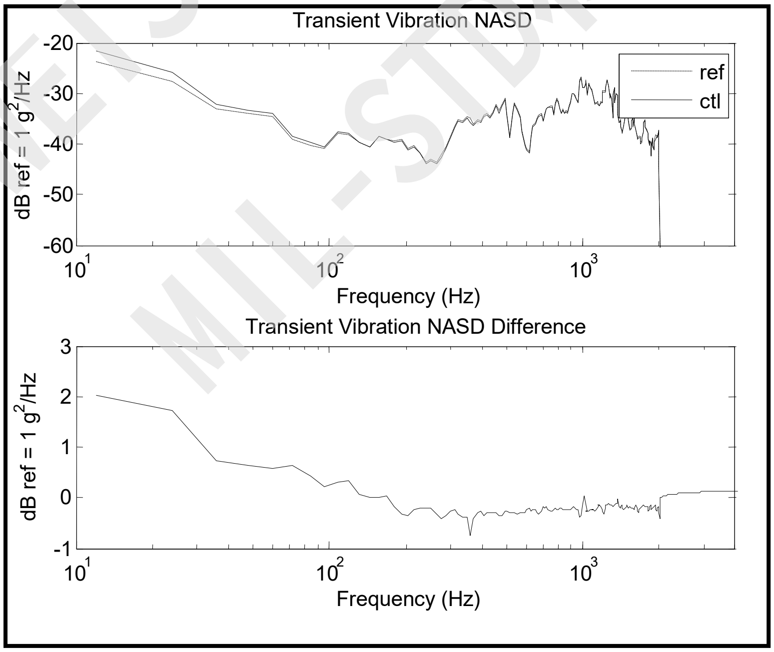


Figure 525.2A-14. Composite normalized ASD estimates for r and c.

From the above statistics, it can be concluded that no valid distinction can be made between r and c under the product model assumption, even though the non-Gaussian distribution of error s is difficult to interpret. It would appear that tolerance for this particular segment could be established as less than 0.2 grms amplitude for 90 percent of the time trace envelope, and 2 dB for the normalized ASD estimates, based on the information in Figures 525.2A-13 and 525.2A-14. This concludes replication error processing and tolerance specification for the transient vibration sub-event.

7. TPP STATIONARY VIBRATION.

Figure 525.2A-15 displays the stationary vibration time traces to be processed for replication error assessment. Note the time trace s is nominal, and that r and c could follow a product model formulation as above because of the comparatively small envelope variation in time.

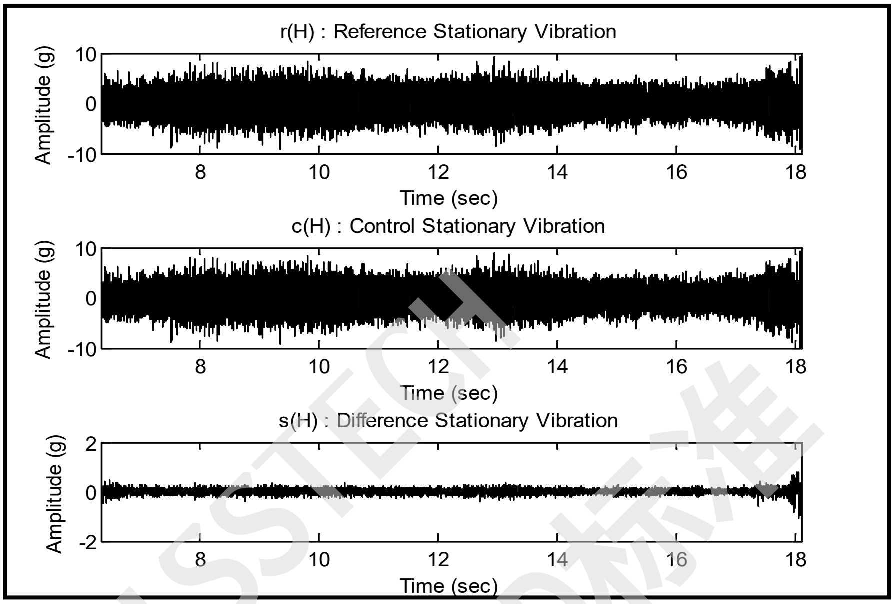


Figure 525.2A-15. Stationary vibration time traces - r, c, and s.

The replication error is assessed under the stationary random vibration assumption as follows:

(1) Probability density estimates are generated for r and c.

(2) s qq-plot is generated to examine the difference time trace for normality.

(3) Fraction-of-Time (FOT) distribution for s

(4) ASD estimates are determined for r, c and s.

To examine the Gaussian form of the stationary vibration trace, the composite histogram (probability density function estimate) for r and c is plotted in Figure 525.2A-16, with the tail behavior enhanced. The time trace information is long-tailed because of the presence of the time-varying mean-square amplitude. “G” represents the Gaussian histogram on the plot legend.

Figure 525.2A-17 provides a qq-plot for s for Gaussian quantiles. The tail behavior of s would seem to indicate that the peak and valley values are somewhat larger than and smaller, respectively, than a Gaussian. Even though the Gaussian portion (good fit to straight line is greater than in the transient vibration case).

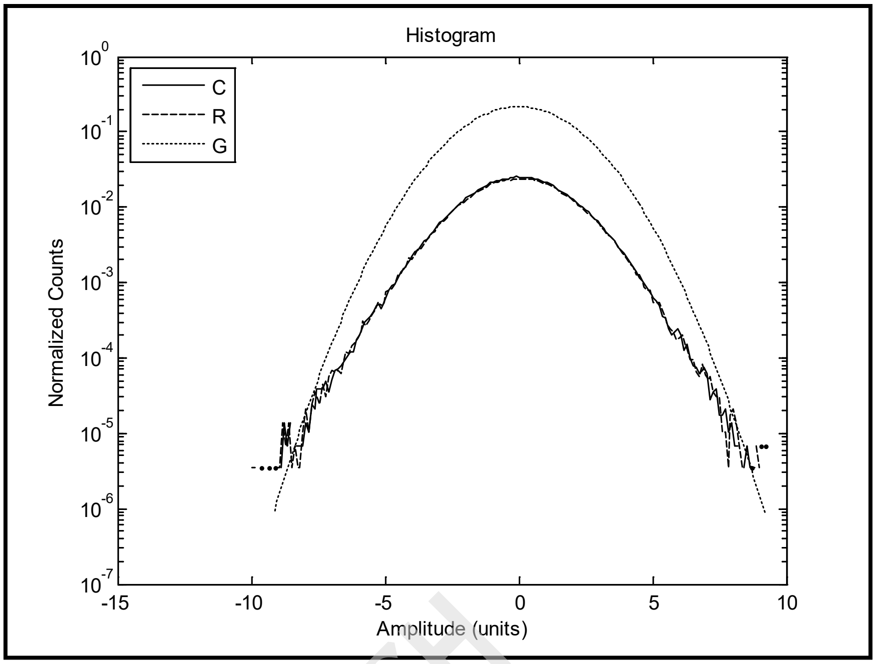


Figure 525.2A-16. Stationary vibration probability density function estimates.

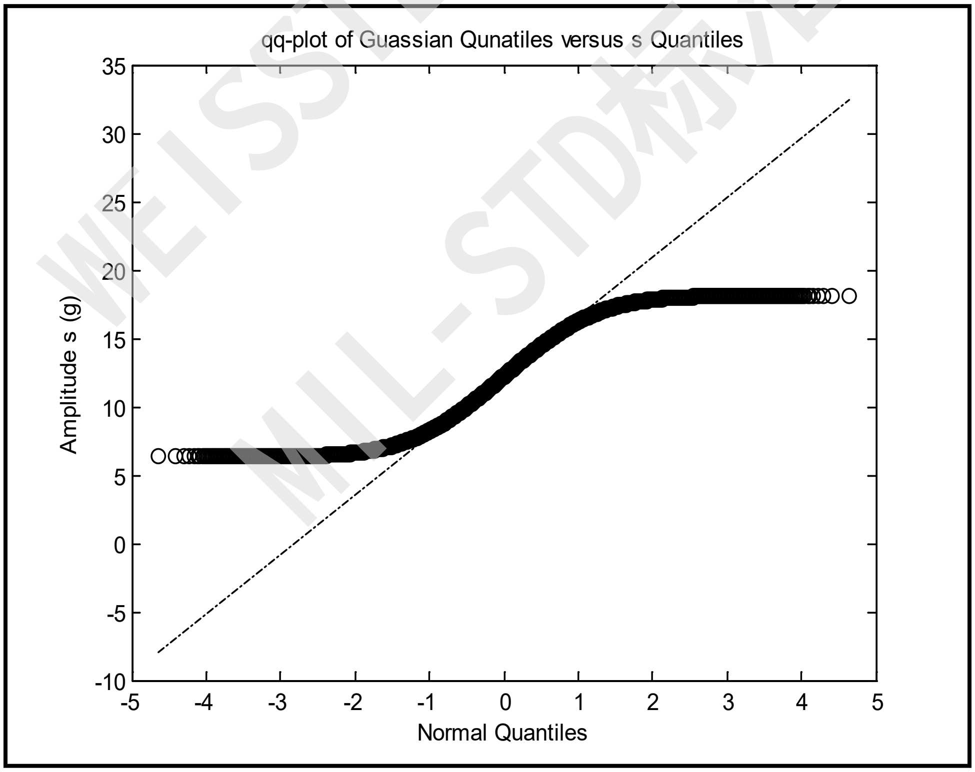


Figure 525.2A-17. Stationary vibration q-q plot for s versus Gaussian.

Annex B defines the FOT distribution for difference time trace assessment. This assessment empirically defines the fraction of time the error lies outside (or inside) given error amplitude bounds. This assessment is mathematically equivalent to a probability density (or distribution) assessment but more transparent and easier to interpret for an allowable error tolerance specification. Since TWR is time based, an allowable error of x-percent of the time the error amplitude may exceed y-percent of the root-energy-amplitude level (REA) of the deterministic reference time trace is easily visualized. Figures 525.2A-18a,b,c display the time-varying error in g's for the stationary segment along with the REA percentage error plotted against the FOT quantiles. For the example under consideration the REA for the reference is 1.85 g-rms. Both two-sided and one-sided analyses are considered. The FOT ranges from 0.0 to 1.0 over approximately plus and minus 10% of the REA. Figure 525.2A-18a displays FOT quantiles for 10% to 10% REA error percentage. Figure 525.2A-18b displays the REA random error -5% to 5% for FOT quantiles from approximately 0.1 to 0.9 and Figure 525.2A-18c considers one-sided error for 10% REA error percentage and the 0.90 FOT quantile. A two-sided tolerance specification might, for example, require not more than 10% (0.10 FOT quantile) of test time to lie outside the REA amplitude percentage bounds of -5% and 5%. Tolerance is in terms of what percentage of time is the error allowed to be larger than a certain percentage of REA as a reference amplitude.

In Figure 525.2A-19, a composite of the ASD estimates for r and c is provided. The ASD estimates between r and c are essentially equivalent. For time trace s, there is non-flat spectrum that normally would not be present if the replication error were of a strong Gaussian character, i.e., s was band-limited white noise. The processing parameters are an analysis bandwidth of 5 Hz applying a Hamming window with 50 percent overlap.

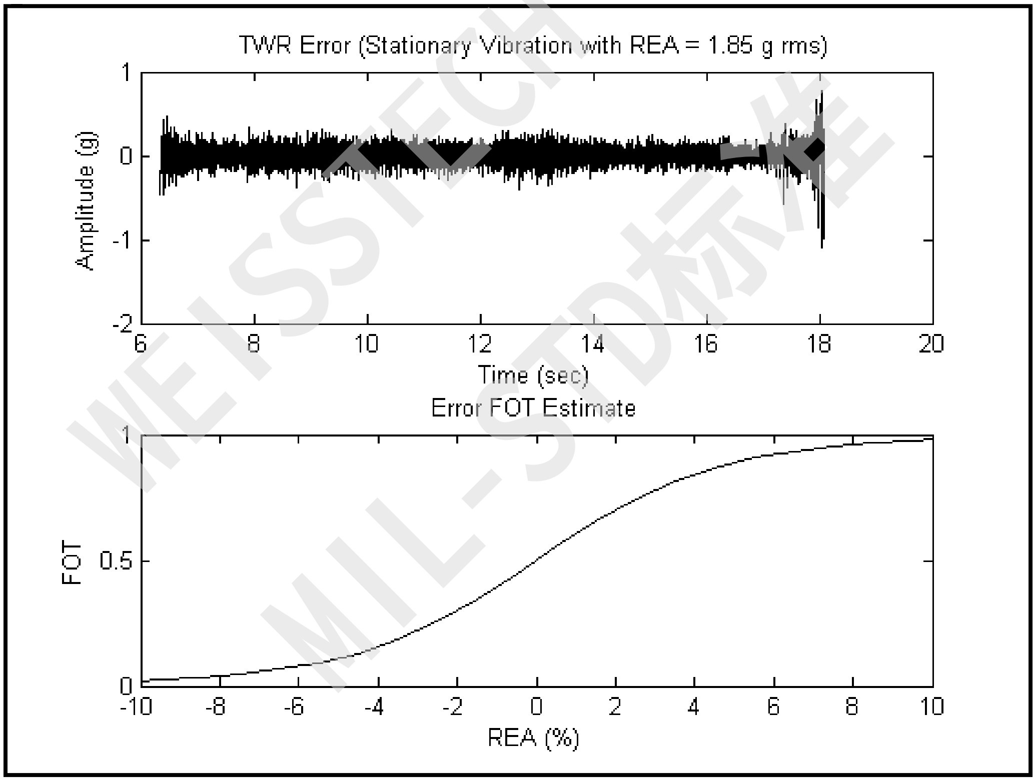


Figure 525.2A-18a. FOT Error Assessment – 10% REA Error Fraction-of-Time (FOT)

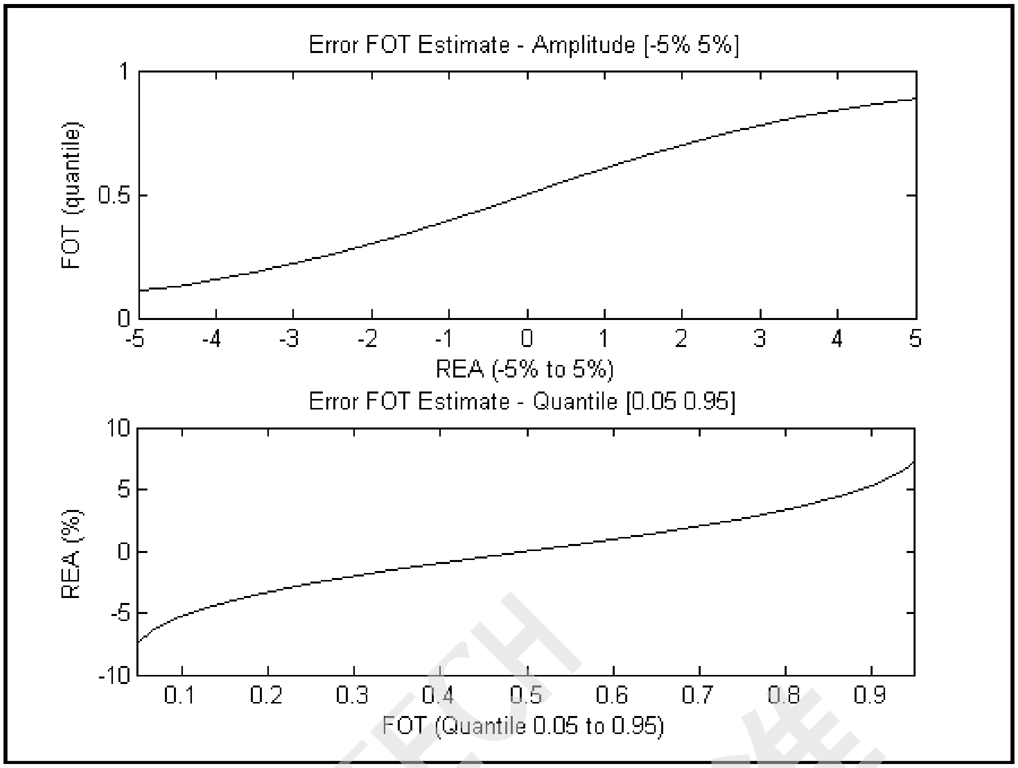


Figure 525.2A-18b FOT Error Assessment - 5% REA FOT Error Bounds

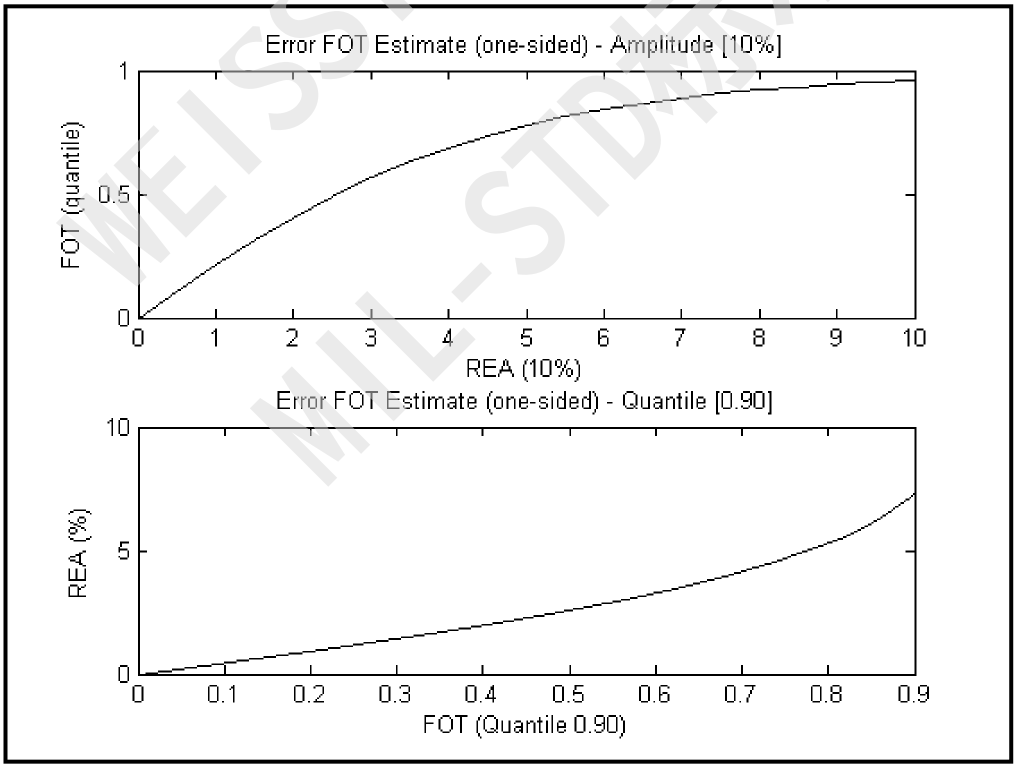


Figure 525.2A-18c FOT Error Assessment - One-sided 10% REA FOT Error Bounds

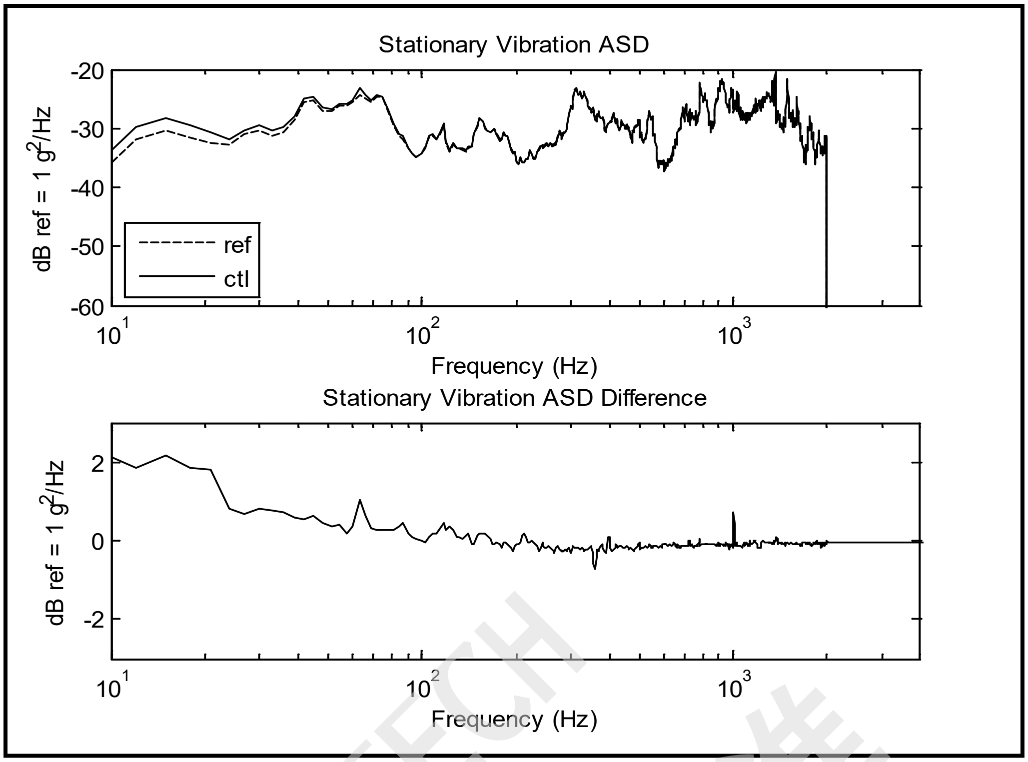


Figure 525.2A-19a. Composite ASD estimates for r and c.

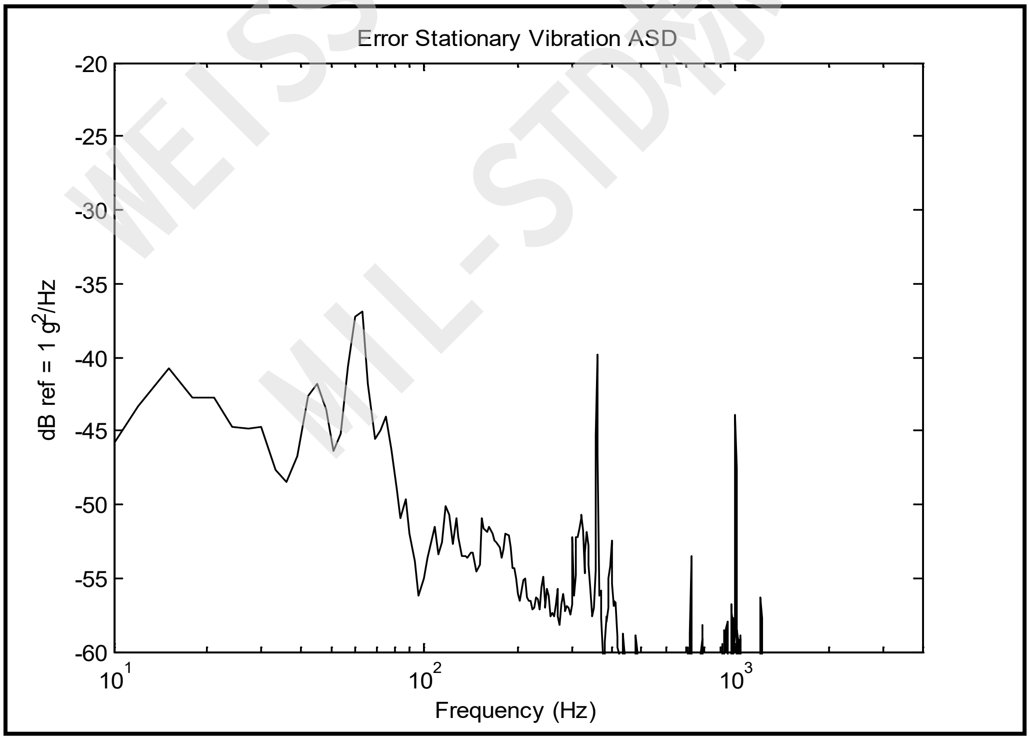


Figure 525.2A-19b. ASD estimate for s.

From the above statistics, it might be concluded that no valid distinction can be made between r and c under the stationary model assumption even though the non-Gaussian distribution of error s is difficult to interpret. It would appear that tolerances for this particular segment could be established as maximum 2 dB for the ASD estimates, based on the information in Figure 525.2A-19. This concludes replication error processing and tolerance development for the stationary vibration sub-event.

8. TPP SHOCK.

Figure 525.2A-20 displays the shock time traces that will be processed for replication error assessment. Note that time trace, s, is not nominal in the area of maximum shock. The maximum/minimum values for each trace are given by r: -22.84/35.24; c(H): -24.28/39.76; and s(H): -4.11/4.78.

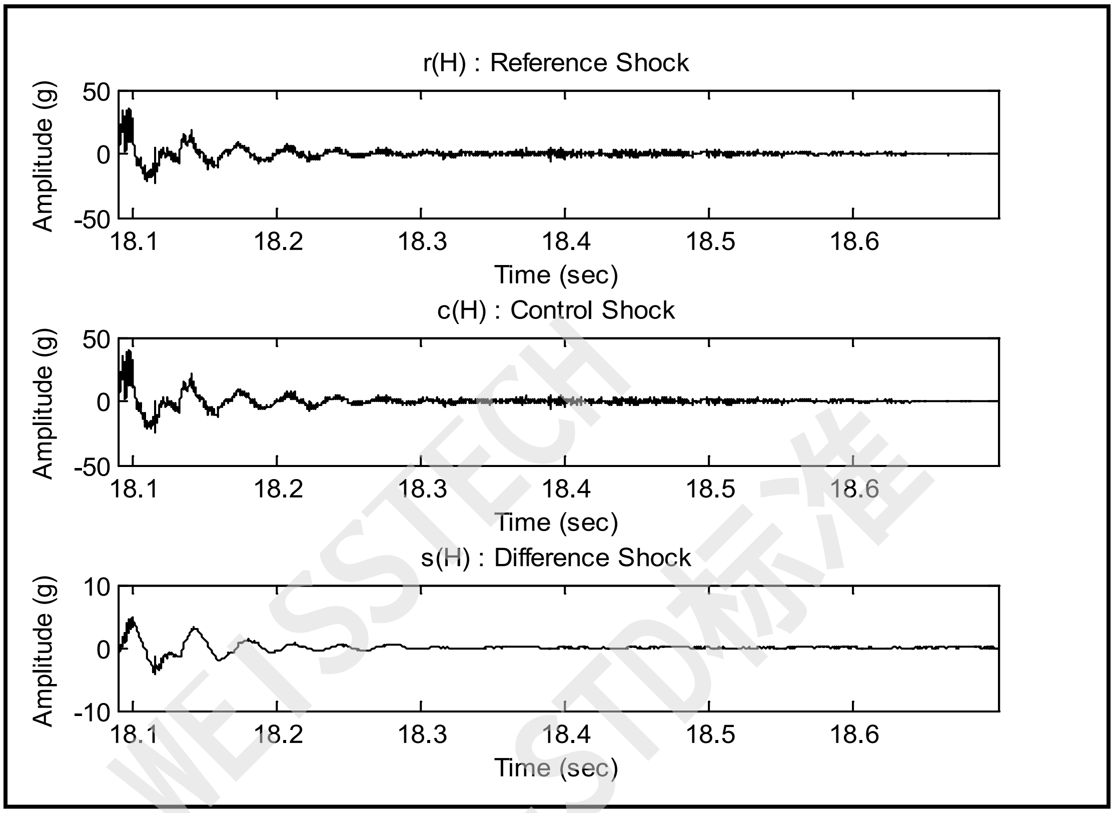


Figure 525.2A-20. Shock time traces - r, c, and s.

The replication error is assessed under the shock assumption as follows:

a. An r versus c cross plot is generated.

b. s qq-plot is generated to examine the difference time trace for normality.

  (1) Pseudo-velocity SRS assessment for r and c.

  (2) ESD estimates are determined for r, c, and s under a shock time trace assumption.

For the shock segment, a cross plot of r versus c provides useful information with regard to the positive and negative peaks. However, from the form of the r and c time traces, it is obvious that histograms and empirical q-q plots versus the Gaussian will yield little useful information. Figure 525.2A-21 provides a cross-plot of r versus c.

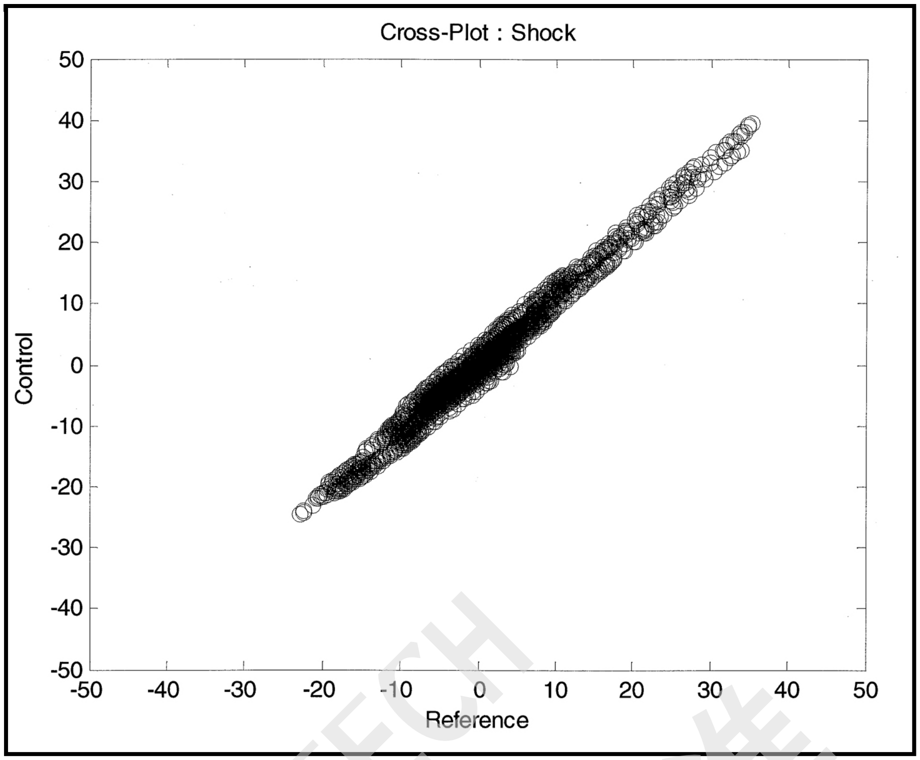


Figure 525.2A-21. r versus c cross-plot.

Even though "s" will not display Gaussian character, some indication of its non-Gaussian character can be useful. Figure 525.2A-22 provides a q-q plot of s versus the Gaussian distribution. Clearly, the sample quantiles from "s" in the tails far exceed any Gaussian model that can be fit to s.

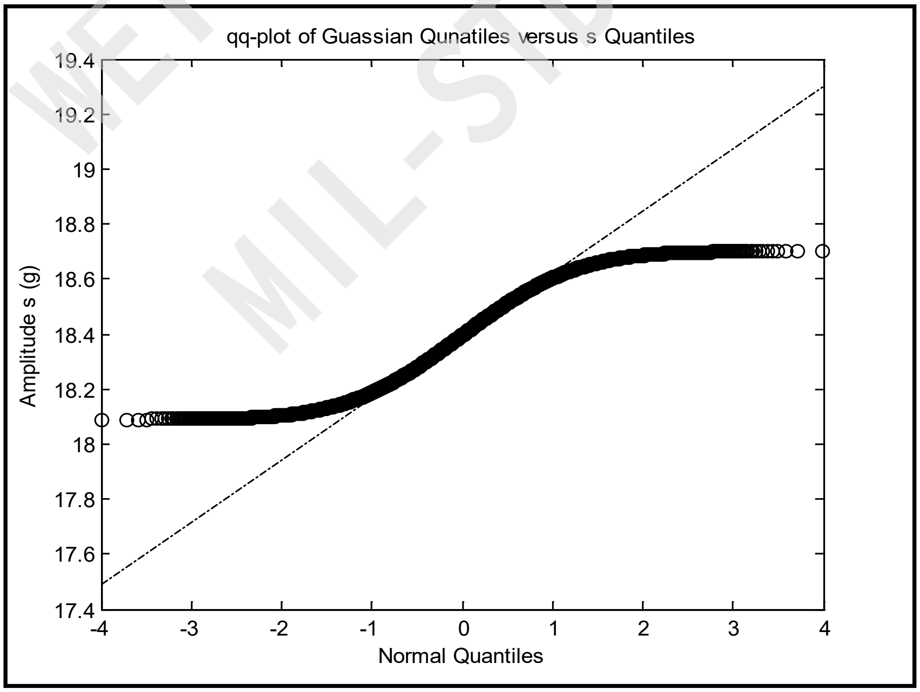


Figure 525.2A-22. Shock q-q plot for s versus Gaussian.

A common way of comparing shock information is through the SRS, in particular the recommended pseudo-velocity SRS estimate (Method 516.8). For the r and c time traces, a composite overlay of the pseudo-velocity SRS estimates for both shocks is useful. Figure 525.2A-23 provides this comparison in addition to a maximax acceleration SRS comparison. Since the SRS is an integration/smoothing process, it is expected that the reference and control information will be highly correlated when viewed in an SRS format. For these figures no wavelet correction was attempted for low frequency correction since such a correction applied individually may lead to a less transparent comparison.

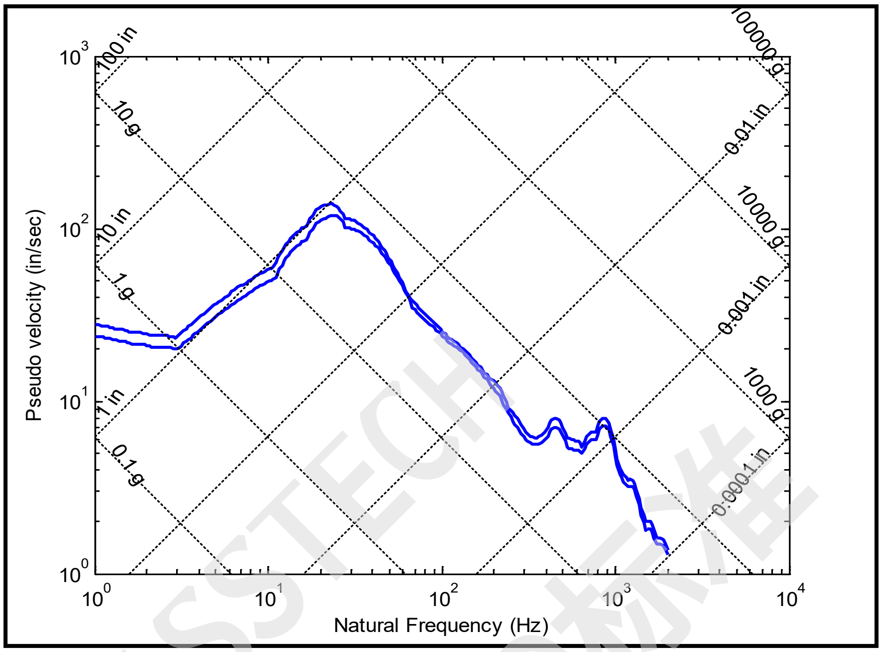


Figure 525.2A-23a Composite pseudo-velocity maximax pseudo-velocity SRS for r and c.

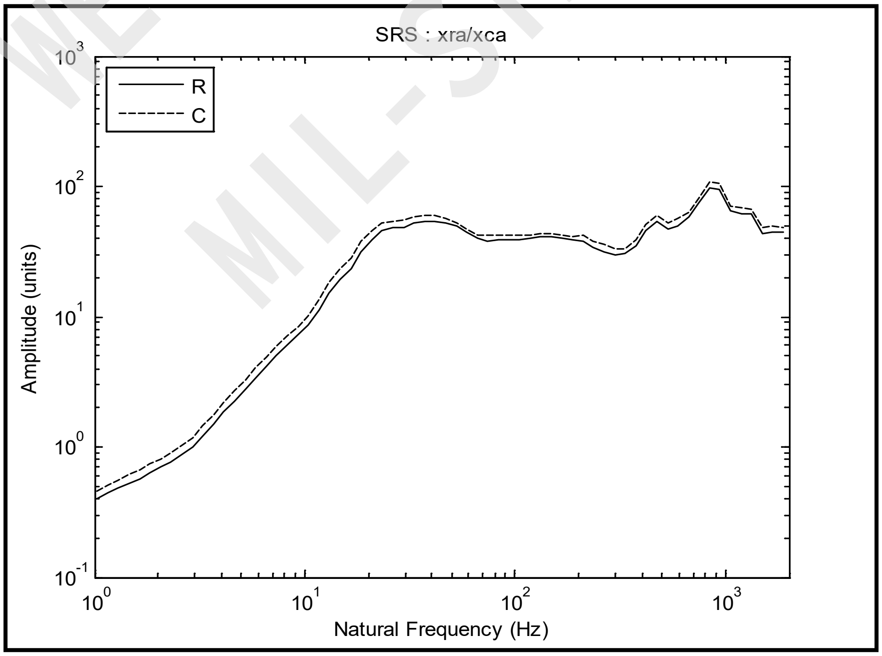


Figure 525.2A-23b. Composite maximax acceleration SRS for r and c.

Since ESD estimates provide a way of comparing shock type events, Figure 525.2A-24 provides a composite of r and c ESD estimates, while Figure 525.2A-25 provides the ESD estimate for "s." It is clear from both of these plots that the most substantial error is found in the low frequency region. This is not surprising since the transfer function used to compensate the entire time trace was likely not optimal for the shock.

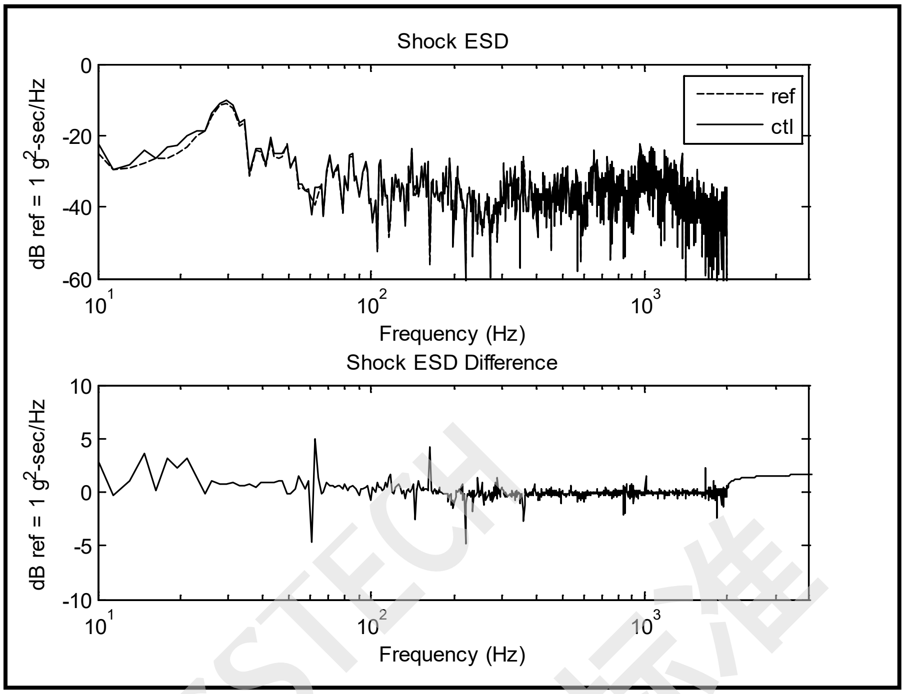


Figure 525.2A-24. ESD estimates for r and c.

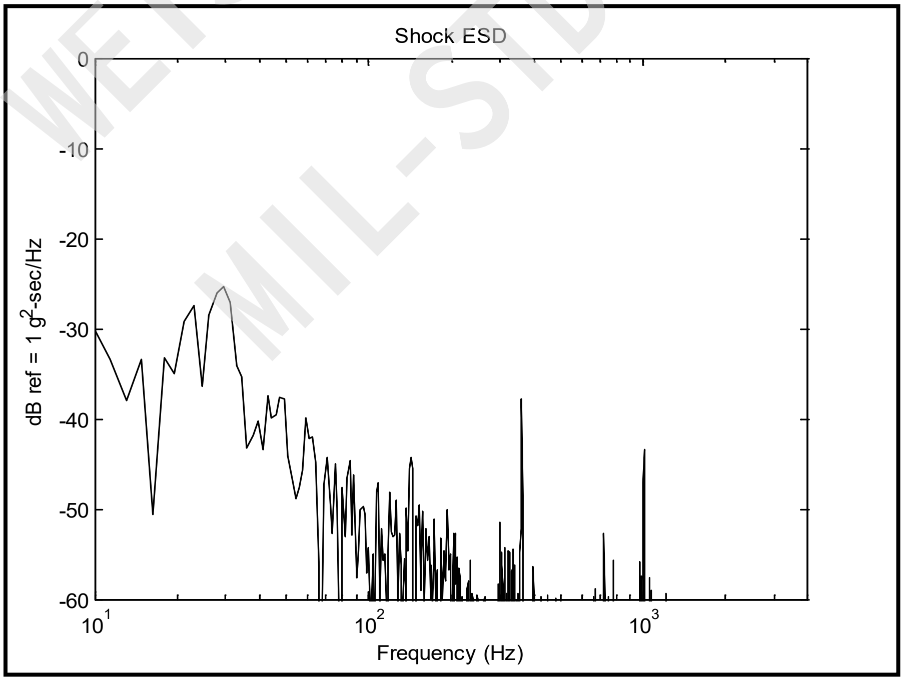


Figure 525.2A-25. ESD estimate for s.

9. POST-TEST PROCESSING FOR STA.

TPP replication error assessment is most stringent for specifying tolerance criteria being that the tolerance criteria must be satisfied for the correlated time points, point-by-point. Replication error averages for STA is most easily defined for application to s, as opposed to application to r and c individually, and then seeking to compare STA r estimates with STA c estimates. Annex B discusses some complications with individual STA application. For Annex A post-test processing, using STA directly centers upon the statistical characteristics of s under short-time averaging. Figures 525.2A-26 and 525.2A-27 display short-time averaging for the mean and root-mean-square of time trace s over the entire time trace displayed in Figure 525.2A-3d-f for 0.05 and 0.20 second averaging times. An averaging time of 0.05 seconds for a bandwidth of 2000 Hz provides 5 percent normalized random error in the root-mean-square estimate, and an averaging time of 0.20 seconds for the SESA bandwidth provides a 5 percent normalized random error in the mean-square estimate. For AC coupled instrumentation measurements, the short-time average mean is near zero - not particularly meaningful, but is computed for completeness. It is clear from these figures that the rate of change of the time trace is too great in the transient vibration, and shock tails of the time trace to provide meaningful estimates by averaging in time. Thus, tolerance information in these two tails requires another basis, e.g., TPP.

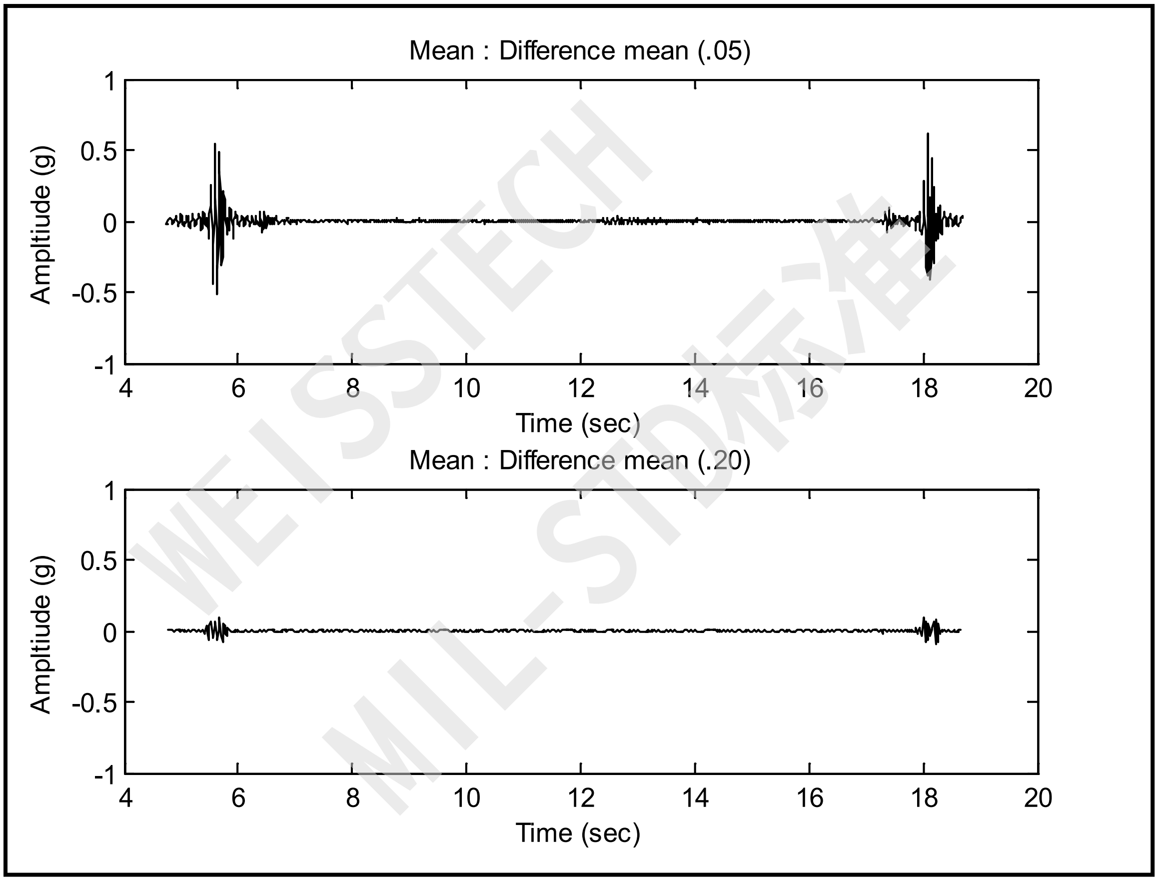


Figure 525.2A-26. Short-time averaging for difference mean.

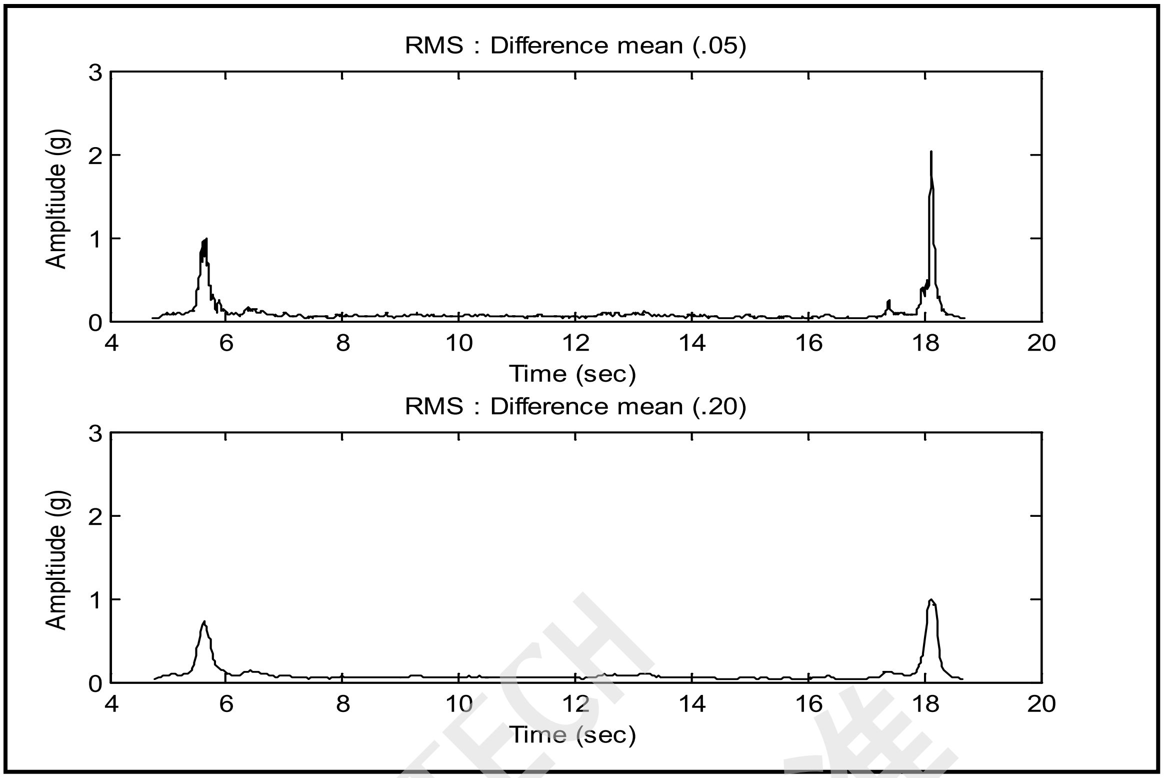


Figure 525.2A-27. Short-time averaging for difference root-mean-square.

Justification for using short-time average estimates for error assessment is that for stationary random processing, the principal comparison with the ASD estimate in the frequency domain is an average, and for shock processing, the principal comparison with the SRS estimate in the single-degree of freedom natural frequency domain is an integrated (or averaged) nonlinear type estimate. Annex B defines time average estimates in continuous form, and in digital form for a rudimentary description of the underlying non-stationary random process. The averaging time is arbitrary, but generally will be such that the normalized bias error is a minimum, and the normalized statistical error in the root-mean-square estimate under Gaussian assumptions is no more than 0.05. The expressions for the normalized root-mean-square error and normalized mean-square error are provided in Annex B.

This concludes Annex A and processing of selected information supplied for SESA TWR. As technology evolves, the information in this Annex will also evolve. Significant evolution needs to take place in understanding the extent of signal compensation, how it is performed, what its limitations are, and just general overall TWR control strategy understanding. This evolution will feed directly into the development of realistic tolerance limits based upon replication error assessment.

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METHOD 525.2, ANNEX B
SUMMARY OF POST-TEST ANALYSIS PROCESSING PROCEDURES AND TEST TOLERANCE SPECIFICATION

1. INTRODUCTION.

The purpose of this Annex is to provide an informational basis for establishing tolerance assessment for single-exciter/single-axis (SESA) time waveform replication (TWR) laboratory tests independent of the vendor software. This Annex does not recommend any single methodology for TWR error assessment and is somewhat incomplete in that the statistical analysis of reference/control scatter plots is not discussed in detail. Correct understanding of reference/control scatter plots, perhaps in linear regression terms, and the accompanying statistics would seem to lie at the heart of TWR test tolerance assessment. In paragraph 4 of this Annex a test tolerance rationale is provided. In the future, vendors may incorporate such tolerance assessment options for the convenience of the test laboratory and determination if test specifications are satisfied. For now test tolerance assessment relative to a specification beyond the vendor software will require a trained analyst and off-line processing of digital sequences through custom software, e.g., MATLAB, LABVIEW, etc. Paragraph 2 provides standard terminology for SESA TWR. The formulas in paragraph 3 may assist in the design of custom software. This Annex does not summarize vendor assessment for replication error. In general, a vendor provides an estimate of the comparison between the reference and control time traces based upon time averaging over a specified time history segment. This time averaging generally takes no account of the form of the time trace, is performed in order to assess error as the test progresses in time (probably for control issues), and provides a rationale for aborting the test if the error exceeds certain prescribed limits. However, since vendor software is fundamental to test control this blocksize should be noted and considered the maximum block size to be used in post-processing error assessment under short-time-averaging (STA).

This Annex assumes that the “reference” time trace is band limited and of a deterministic in nature even though it may be a sample time trace from a field measured random process. This Annex assumes that the “control” time trace is stochastic in nature. This defines a SESA model whereby a deterministic time trace is input to a “random system” that provides a stochastic output. The randomness of the system comes from all the unquantified details of the reproduction of the deterministic input time trace including boundary conditions, compensation, system noise etc. The distinction between a “deterministic” and a “stochastic” reference time trace is subtle. The easiest way to visualize this distinction is to think in terms of a regression model for which there is an independent variable selected ahead of time and a dependent variable that reflects a dependence upon the value of the independent variable. In data analysis when both variables are associated the relationship between them is a “structural” relationship as opposed to a “regression” relationship since both variables in the “structural” relationship are subject to estimation and random error. A second subtle feature of the processing is that a “statistical basis” as opposed to a “probabilistic basis” is assumed. The statistical basis allows for “time averages” as opposed to requiring “ensemble averages” for a probabilistic basis. This seems natural since seldom is it useful to consider SESA TWR reference and control time traces in terms of ensembles.

In description of the assessment to follow, this Annex assumes that the bandwidth for comparison i.e., error between the reference trace, $r(t)$, and the control time trace, $c(t)$, is comparable. It is important that the test personnel understand clearly the bandwidth of all time traces from field measurement, unprocessed control time trace and the error time trace, $s(t)$, defined below. See Annex A paragraph 4.2 for a more detailed discussion of time trace band limit considerations.

2. TERMINOLOGY.

In this Annex ***replication error assessment*** or equivalently ***test tolerance assessment*** refers to examining the properties of the difference (as a function of time) between the TWR” input” and the TWR “output”. TWR ***“test specification”*** refers to using the results of the error assessment to determine if the laboratory TWR test replicated the “input” satisfactorily i.e., if ***“test tolerances”*** common to other Methods are satisfied for TWR. For Method 525 there are potentially five categories related to test specification.

In this paragraph, the continuous analog time traces are represented by lower case letter as a function of time, t. The upper case associated letters represent the random variables obtained by sampling the properly signal conditioned analog time traces. The TWR reference time trace, $r(t)$, is considered to be band limited and deterministic in nature. It is specified in an ASCII file with required oversampling for replication. The TWR control time trace, $c(t)$, is stochastic as a function of the test configuration including compensation strategy and system noise. The difference between the control and reference time traces, *s(t)*, is stochastic in nature and is the primary time trace to be used in the TWR error assessment and tolerance specification.

For *R* deterministic and *S* and *C* stochastic variables and a physical correspondence between *r(t)* and *c(t)*, i.e., *c(t)* output resulting from TWR then define

(1) *R* associated with $r(t)$ as $R = \{r[n], n = 1, 2, ..., N\}$

(2) *C* associated with $c(t)$ as $C = \{c[n], n = 1, 2, ..., N\}$ and

(3) *S* associated with $s(t) = c(t) - r(t)$ $S = \{s[n] = c[n] - r[n], n = 1, 2, 3, ..., N\}$

If the two continuous time traces *r(t)* and *c(t)* are identical according to "*time-point by time-point*" (TPP), then the time trace represented by the reference time trace has been replicated exactly in the laboratory. Generally the reference and control time traces are not TPP identical and "statistics" must be introduced to quantify *s(t)*. Stochastic *S* has no preconceived theoretical probability distribution function (in fact *s(t)* or *S* provides an "optimum" estimate for error assessment in the sense that the statistics of gross averages are of lesser importance in error assessment. As has been demonstrated in Annex A, *S* is generally neither Gaussian distributed nor stationary. Once *S* has been determined and parameters of *R* known, *R* and *C* will play a lesser role for tolerance assessment except for Category III and Category IV specification in paragraph 4.

3. REPLICATION ERROR (TEST TOLERANCE) ASSESSMENT EXPRESSIONS.

For replication error assessment, it may be useful to ***nonuniformly*** time weight or "window" *s(t)* over a time interval before making error estimates but the rationale for such weighting is beyond the scope of discussion here . For the replication error assessment to follow, two options are available:

(1) examining the statistical properties of sequence *S* directly in an overall or "global" sense

(2) examining sequence *S* under "*short-time averaging"* (STA) yielding stochastic variable S_A for statistical assessment where S_A represents a "local" average and the total set of "local" averages summarizes *S*

The stochastic estimates S_A have bias error and random error, but it is assumed that judicious selection of the "window" has representative random error and minimum bias error.

The time averaging procedure can be applied to ***functions of*** *s(t)* such as the instantaneous mean-square level of *s(t)*, i.e., $s^2(t)$. In using STA for replication error assessment, the summary statistics need to be clearly defined, and any note made of dependence introduced in the averaging process e.g., serial correlation of shifted average values.

Since it is assumed that for $E\{\ \}$ the expectation operator on stochastic variables and $S = C - R$, then

$$E\{S\} = E\{C - R\} = E\{C\} - R \,.\, E\{S_A\} = E\{\bar{S}\} = E\{\overline{(C - R)}\} \approx E\{\bar{C} - \bar{R}\} = E\{\bar{C}\} - \bar{R} = E\{\bar{C}\} - R_A = C_A - R_A \,.$$

Replication error assessment precedes TWR tolerance specification, however replication error assessment must relate directly to tolerance specification. For example, tolerance specification for TWR is not viable for "single point" error assessment i.e., maximum of *S* but maximum of *S* may be a meaningful parameter. In addition the deterministic reference, *R*, is generally oversampled by a factor of ten or more based upon TWR requirements. It is safe to assume that a "nominal window" for error assessment could be a uniform time interval with the number of points equal the oversample factor. This implies that "smoothed" error estimates applied to sequence *S* are fundamental in replication error assessment and subsequent tolerance specification. As noted above generally the smoothing window should not exceed the vendor control blocksize. The oversample factor and this blocksize provide bounds on STA averaging time selection.

In the expressions to follow, processing will take place over a uniform time interval $T = [T_{i+1} - T_i]$. Formulas provided will be expressed in a continuous form followed by a discrete digital form. In general, the error statistics for the estimators will be provided for the ideal case in which *s(t)* is bandwidth limited white noise of bandwidth *B*. The role the error statistics for the estimators play is to insure that artificial estimation errors in replication error assessment are minimal when compared to the replication errors to be used in tolerance specification. As mentioned above, seldom is the character of *s(t)* so simple, so that the processing error statistics are approximate for other than bandwidth limited white noise. Normalized random errors are provided for most estimates. Bias error occurs whenever averaging takes place, however for averaging windows on the order of the oversample factor bias error should be minimal. Whenever practical bias errors in the estimates for the error assessment need to be minimized. If there exists questions relative to the size of normalized bias and random errors, much more detailed processing beyond the scope of this Annex may need to be employed (paragraph 6.1, reference a).

In description of the error assessment expressions, the designation "local" or "global" is made. The term "local" refers to a statistic that is useful for processing short segments of time-varying traces, while the term "global" refers to a statistic that is better suited to summarizing overall time traces. For example, the collection of STA for ***S*** root-mean-square provides "local" estimates related to a potential tolerance specification. The cumulative probability density function estimate for *S* describes error as being perhaps Gaussian or non-Gaussian. This is a "global" assessment from which a tolerance specification might be based upon the distributional form of the estimate. Generic variables

$x(t)\ (x[n], n = 1,2,...,N)$, $y(t)\ (y[n], n = 1,2,...,N)$ and $z(t)\ (z[n], n = 1,2,...,N)$

are employed in the formulas whereby *r(t)*, *c(t)*, and *s(t)* may be substituted at will depending upon interpretation. In the formulas to follow *M* will be an "index" related to the time sample interval for the time average estimate (it is a time shift parameter for averaging) and N_a will be the number of time points averaged over. $\lfloor N_a/2 \rfloor$ is the greatest integer designation for $N_a/2$. It is assumed that $M = \lfloor N_a/2 \rfloor + \lfloor N_a/2 \rfloor - 1$ where generally M is an odd number to prevent any phase shift introduced in the processing.

There are three cases in which joint consideration of deterministic *R* and stochastic *C* may be useful. In the first case a scatterplot constructed by plotting the point $(r(n), c(n))$ in the plane will reveal valuable information relative to a single plot of the error *s(n)*. In the second case since computation of an ASD/ESD estimate over a deterministic time trace has some meaning the comparison of the ASD/ESD estimates for *r(n)* and *c(n)* may provide meaningful information in relation to the ASD/ESD for *s(n)*. In particular the deterministic estimate is divided into the stochastic estimate to examine the ratio in the frequency domain. Finally, comparison of SRS estimates for *r(n)* versus *c(n)* along with an SRS estimate for *s(n)* i.e., the "noise" can be useful.

For easy reference the following table is provided:

Table B-I. Summary of error assessment expressions

E1	MEAN (local & global) *S*
E2	ROOT-MEAN-SQUARE & MEAN-SQUARE (local & global) *S*
E3	COVARIANCE, CORRELATION and SCATTER-PLOT (global) *R* and *C*
E4	PROBABILITY DENSITY,CUMULATIVE PROBABILITY and QUANTILE (global) *S*
E5	FRACTION-OF-TIME (global) *S*
E6	ASD/ESD/PERIODOGRAM (global) *R* and *C*
E7	SHOCK RESPONSE SPECTRA (global) *R* and *C*

Expressions E1 through E7 are potentially useful for TWR tolerance specification. Future editions of MIL-STD-810 will likely refine and add to these expressions as SESA TWR testing becomes more common and experience with both replication error assessment and subsequent test specification becomes more common. Generally E1, E2 E5, E6, and E7 will directly relate to tolerance specification. E3 and E4 provide good qualitative information but will not directly relate to tolerance specification.

E1: MEAN (local & global)

A collection of STA for *s(n)* provides an indication of any potential “shift” in very low frequency information contained in *r(t)* under TWR. A zero mean error is desirable otherwise bias may be present. The mean estimate for $x(t)$ is defined as follows:

$$\hat{\mu}_x(t_i) = \int_{T_{i-1}}^{T_i} x(t)dt \leftrightarrow \hat{m}_{x_i} = \frac{1}{N_a} \sum_{i=M-\lfloor N_a/2 \rfloor+1}^{M+\lfloor N_a/2 \rfloor} x(t_i) \tag{1}$$

The normalized random error in the mean estimate in units of root-mean-square is defined as

$$\varepsilon[\hat{\mu}_x] \approx \frac{1}{\sqrt{2BT}}\left(\frac{\sigma_x}{\mu_x}\right) \text{ for } \mu_x \neq 0,\ B \text{, overall bandwidth of } x(t) \text{, and } T \text{, averaging time.} \tag{2}$$

Note that this is related to the confidence interval with confidence coefficient $1-\alpha$

on the mean of a population (not necessarily a time history) obtained by a sample of size N i.e.,

$$CI_{\mu_x;1-\alpha} = \left[\bar{x} - \frac{\sigma_x z_{\alpha/2}}{\sqrt{N}} \le \mu_x \le \bar{x} - \frac{\sigma_x z_{\alpha/2}}{\sqrt{N}}\right].$$

E2: ROOT-MEAN-SQUARE and MEAN-SQUARE (local & global)

A collection of STA root-mean-square levels in time is fundamental for replication error assessment and probably is closely aligned with vendor TWR error assessment. It is basically a “rms” error. The mean-square error assessment is included for completeness but is generally not particularly useful.

The root-mean-square of $x(t)$ with zero mean over a short interval of time is computed as follows:

$$\hat{\psi}_x(t_i) = \sqrt{\int_{T_{i-1}}^{T_i} \left[x(t) - \mu_{x_i}\right]^2 dt} \leftrightarrow x_x(t_i) = \sqrt{\frac{1}{N_a - 1} \sum_{i=M-\lfloor N_a/2 \rfloor+1}^{M+\lfloor N_a/2 \rfloor} \left[x(t_i) - m_{x_i}\right]^2} \tag{3}$$

and the normalized random error for the root-mean-square estimate is given by,

$$\varepsilon[\hat{\psi}_x] \approx \frac{1}{2\sqrt{BT}} \text{ for } B \text{, overall bandwidth of } x(t) \text{, and } T \text{, averaging time.}$$

This estimate is essentially an estimate of the standard deviation of the time trace over a short time interval.

The mean-square of $x(t)$ with ***zero mean*** over a short interval of time is computed as follows:

$$\hat{\psi}_x^2(t_i) = \int_{T_{i-1}}^{T_i} x^2(t)dt \leftrightarrow std_x^2(t_i) = \frac{1}{N_a} \sum_{i=M-\lfloor N_a/2 \rfloor+1}^{M+\lfloor N_a/2 \rfloor} x^2(t_i) \tag{4}$$

For overall bandwidth B in Hz and averaging time T in seconds, the normalized random error for the mean-square estimate is given by

$$\varepsilon[\hat{\psi}_x^2] \approx \frac{1}{\sqrt{BT}}. \tag{5}$$

This estimate is essentially an estimate of the variance of the time trace over a short time interval.

That is the confidence interval with confidence coefficient $1-\alpha$

on the standard deviation of a population (not necessarily a time history) obtained by a sample of size N, i.e.,

$$CI_{\sigma;1-\alpha} = \left[\frac{ns^2}{\chi^2_{n;\alpha/2}} \leq \sigma_x^2 \leq \frac{ns^2}{\chi^2_{n;1-\alpha/2}} \right] \text{ for } n = N\text{-}1 .$$

For application for $B = 2000Hz$ and $T = 0.01$ or 0.1 seconds the normalized random error for a mean comparable to the standard deviation, root-mean-square and mean-square is 0.16, 0.11, 0.22 respectively for averaging time of 0.01 seconds, and 0.05, 0.04, 0.07 respectively for averaging time of 0.1 seconds. To obtain a meaningful characterization of *x(t)*, it is important the normalized random error be minimized by as long an averaging time as is consistent with nominal bias error.

E3: COVARIANCE, CORRELATION, and SCATTER-PLOT (global and local)

Generally, covariance and correlation can be viewed as meaningful in the case of regression between a deterministic and a random time trace i.e., *r(t)* and *c(t)* Since *s(t)=c(t)-r(t)* no new information is added by computing the correlation or covariance between *r(t)* and *s(t)*. Covariance and correlation should be viewed in terms of a "regression fit" of *r(n)* to *c(n)*. This particular replication error assessment is somewhat qualitative thus not particularly useful for tolerance specification e.g., specifying a correlation coefficient for tolerance would be too gross a parameter to be meaningful. The covariance relationship between two time traces over a short interval of time (local covariance), or over the entire time trace (global covariance) is computed in the time domain as follows:

$$\operatorname{cov}(x,y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y}) \tag{6}$$

This quantity can be normalized to provide the local or global correlation coefficient that can be expressed as follows:

$$r_{xy} = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\left[\sum_{i=1}^{N}(x_i - \bar{x})^2 \sum_{i=1}^{N}(y_i - \bar{y})^2\right]^{1/2}} \tag{7}$$

The time trace basis for these expressions from traditional data analysis is as follows. For two arbitrary random processes $\{x_k(t)\}$ and $\{y_k(t)\}$ whose sample functions are indexed on *k* and for which the ensemble means are defined by $\mu_x(t) = E[x_k(t)]$ and $\mu_y(t) = E[y_k(t)]$ where expectation is over index *k* then the cross covariance function at arbitrary fixed values of $t_1 = t$ and $t_2 = t + \tau$ is given by

$$C_{xy}(t, t+\tau) = E\Big[\big(x_k(t) - \mu_x(t)\big)\big(y_k(t+\tau) - \mu_y(t+\tau)\big)\Big]. \tag{8}$$

If $\tau = 0$ then $C_{xy}(t,t) = E\Big[\big(x_k(t) - \mu_x(t)\big)\big(y_k(t) - \mu_y(t)\big)\Big] = C_{xy}(t)$, and this is of the form of the covariance expression above only where the expected value is not over an ensemble indexed on *k*, but over a finite time interval of length $N\Delta t$. The expression for r_{xy} is merely a "normalized" version of the expression for $\operatorname{cov}(x,y)$ defined above. When the k^{th} sample functions $x_k(i\Delta t)$ and $y_k(i\Delta t)$ for $i = 1,2,...,N$ are plotted on the *x* and *y* axes, respectively, the resulting plot is termed a "scatter-plot." The "scatter-plot" depicts the degree of covariance or correlation between two time traces. For r_{xy} in the neighborhood of zero there tends to be no correlation between time traces, and the "scatter-plot" reveals an ellipse with major and minor axes approximately equal. For a distribution of r_{xy} close to either –1 or +1, there is substantial correlation between the time traces, and the "scatter-plot" provides an ellipse with a very small minor axis. In general "scatter-plot" information at the amplitude extremes is of most interest since this defines the correspondence between time trace peaks and valleys.

E4: PROBABILITY DENSITY, CUMULATIVE PROBABILITY, and QUANTILE (global)

A probability density function estimate is generally termed a histogram. A useful indicator of the form of time trace amplitudes is the histogram and its counterpart, the cumulative histogram. Generally, this analysis display is useful only for stationary time traces of substantial duration, e.g., 5 seconds or more. Time traces with even small time-varying root-mean-square levels almost always invalidate this procedure unless some finite distribution mixture can be specified. The histogram is useful usually when it is compared to a theoretical probability density function of an assumed form, e.g., the Normal probability density function. With time trace amplitude bins along the horizontal axis, and "bin counts" along the vertical axis, the logarithm of the bin counts may be used to examine the (1) shape of the histogram for the mid bin ranges, and (2) difference in tails for the small amplitude and the large amplitude bins. Because the probability structure of the difference can be so important in assessing the nature of TWR error, a rather complete discussion of its statistics is provided here. The probability density and probability estimate of $x(t)$ are defined as follows:

From paragraph 6.1, reference a, the probability of *x(t)* taking values between $a-\frac{W}{2}$ and $a+\frac{W}{2}$ during time interval T (where "a" is amplitude level and "W" is a width designation for a time trace amplitude) is estimated as:

$$\hat{P}_x[a,W] = \text{Probability}\left[\left(a-\frac{W}{2}\right) \le x(t) \le \left(a+\frac{W}{2}\right)\right] = \frac{1}{T}\sum_i \Delta t_i = \frac{T_a}{T} \qquad (9)$$

with $P_x[a,W] = \lim_{T\to\infty} \hat{P}_x[a,W] = \lim_{T\to\infty}\frac{T_a}{T}$. The probability density $p_x(a)$ is defined as:

$$p_x(a) = \lim_{W\to 0}\frac{P_x[a,W]}{W} = \lim_{\substack{T\to\infty \\ W\to 0}}\frac{\hat{P}_x[a,W]}{W} = \lim_{\substack{T\to\infty \\ W\to 0}}\hat{p}(a) \text{ where } \hat{p}(a) = \frac{\hat{P}_x[a,W]}{W} = \frac{T_a}{TW}. \qquad (10)$$

From this development, the cumulative probability density and probability density are related as follows:

$$\hat{P}_x[a] = \int_{-\infty}^{a}\hat{p}_x(\xi)d\xi \leftrightarrow \hat{P}_x[a] = \sum_{i=1}^{N}\hat{p}_x[a_i]\Delta a \qquad (11)$$

The normalized mean square random error for the probability density estimate is given from paragraph 6.1, reference a as follows:

$\frac{c^2}{2BTWp_x(a)}$ where, for continuous bandwidth with noise $c \approx 0.3$. Since probability density estimates are particularly susceptible to bias error, the mean square bias error is given as

$$\frac{W^4}{576}\left[\frac{p_x''(a)}{p_x(a)}\right]^2 \cdot \text{for } p_x''(a) \text{ the second derivative of evaluated } p_x \text{ at "a".} \qquad (12)$$

It may be useful to compare the probability structure of $x(t)$ directly to a known probability structure such as the Normal probability density/distribution. This can be done in this formulation by merely plotting the estimated probability structure of $x(t)$ along with the selected theoretical probability structure. There are both parametric and nonparametric statistical tests that allow comparison of probability structures at selected levels of significance. In particular, the nonparametric Kolmogorov-Smirnov test provides a basis for comparison of two sample probability distribution estimates or one sample probability distribution estimate with a theoretical probability distribution estimate. It is possible to use statistical hypothesis testing for purposes of tolerance specification provided the properties of such statistical tests are well understood and such tolerance specification is meaningful.

A strong visual test for equivalence of reference and control distributions is a plot of the quantiles of the two time history trace cumulative distribution probability functions, and is termed a quantile-quantile (q-q) plot. The quantile is defined in terms of the probability distribution function as follows:

For the probability distribution function F with probability density function f, the q^{th} quantile of F, x_q is defined as follows:

$$q_F = \int_{-\infty}^{x_q} f(x)dx \text{ where } 0 \le q_F \le 1 \left(\leftrightarrow q_F \approx \sum_{i=1}^{\hat{x}_q} \hat{f}(x_i)\Delta x_i \text{ where } 0 \le q_F \le 1 \right) \tag{13}$$

and similarly, for the probability distribution G with probability density function g, the q^{th} quantile of G, y_q is defined as:

$$q_G = \int_{-\infty}^{y_q} g(y)dy \text{ where } 0 \le q_G \le 1 \left(\leftrightarrow q_G \approx \sum_{i=1}^{\hat{y}_q} \hat{g}(y_i)\Delta y_i \text{ where } 0 \le q_G \le 1 \right) \tag{14}$$

For a given quantile q, the plot of $\hat{x}_q$ versus $\hat{y}_q$ on a rectangular axis is termed a "$q-q$ plot." F and G may be both analytical, both empirical (estimated from data), or a combination of analytical and empirical.

Examination of the "tails" or extreme values (peaks and valleys) along with the fit to a theoretical Gaussian distribution function, provides the most useful information.

Application of this procedure is most common for plotting the quantiles of the distribution of $s(t)$ against those of the Gaussian distribution function. It is also useful for empirical estimates of $r(t)$ and $c(t)$ against one another, or $r(t)$ and $c(t)$ separately against the Gaussian distribution quantiles. It is important to remember that in all such plots, particularly between $r(t)$ and $c(t)$ time correlation information is lost. It is noted that once the "probability" function of $s(t)$ is established then higher order moments related to skewness or kurtosis can be established.

E5: FRACTION-OF-TIME (global)

Closely related to the probability/quantile amplitude assessment in E4 is the Fraction-of-Time (FOT) assessment. For the FOT estimate of the error is above a certain magnitude and is assessed more intuitively and directly. It is also important to note that for FOT assessment, generally no theoretical distributional form is attached to the FOT estimate e.g., FOT is never spoken of as being Gaussian distributed, etc. For statistical analysis of time series the FOT assessment replaces the more traditional probability analysis., however, FOT distribution is a valid probability distribution function. For processing on a statistical basis the Fraction-of-Time (FOT) is defined as follows:

$$F_T(t;\xi;x) = \frac{measure\{u \in [t,t+T] : x(u) \le \xi\}}{measure\{u \in [t,t+T]\}} = \frac{1}{T}\int_t^{t+T} U(\xi - x(u))du \tag{15}$$

where

$$U(\tau) = \begin{cases} 1 & \tau \ge 0 \\ 0 & elsewhere \end{cases}$$

For the error time trace, *s(t)*, FOT allows assessment of the percentage of time the error is above a certain level and a correct display would indicate the times along the reference time trace *r(t)* for which this occurs. Generally, this is summarized in a single plot similar to the probability based cumulative distribution function estimate. Thus if

$F_T(t;\xi_1;s) \le 0.05$ and $F_T(t;\xi_2;s) \ge 0.05$ then $s(t)$ lies between ξ_1 and ξ_2
ninety percent of the TWR test time where it is assumed ξ_1 and ξ_2
can be related to some level of the reference e.g., the range of the reference,
for purposes of developing a test specification on replication error.

E6: ASD/ESD/PERIODOGRAM (global)

For a deterministic time trace such as *r(t)* a frequency domain estimate is meaningful and similar to the fitting of a Fourier series to an analytically defined function. Visual comparison between frequency domain estimates for *r(t)* and *c(t)* can be made and the ratio of the estimates at each frequency line provided by ratioing the computed quantities (this must never be interpreted as a "transfer function estimate" between the reference and the control time traces).It might be noted that for TWR the "transfer function estimate" is provided in the vendor software in the form of the frequency domain Fourier "drive signal compensation" function. The frequency domain estimates provide for tolerance specification that is directly related to tolerance specifications in Method 514. The basic definition of the windowed two-sided periodogram for an N point digital sequence $\left\{x_t, t = 1,2,...,N\right\}$ in continuous frequency form is as follows:

$$\hat{P}^{(p)}\left(f\right) = \frac{\Delta t}{N}\left|\sum_{t=1}^{N} w_t x_t e^{-i2\pi ft\Delta t}\right|^2 \quad \text{for } -.5 \le f \le .5 \tag{16}$$

Generally the two-sided periodogram is made one sided by multiplying by a factor of 2 with $0 \le f \le 0.5$, and the periodogram is sampled at discrete frequencies, f_i for $i = 0,1,2,...,N/2$ with a uniform spacing of $\Delta f = 1/N\Delta t$. The ASD and ESD can be defined in terms of the sampled periodogram. An ASD estimate is typically a time average sampled periodogram estimate over a limited time interval, with an applied window to reduce spectrum leakage. For stationary time traces the ASD represents a powerful means of comparison between $r\left(t\right)$ and $c\left(t\right)$, and a display of the frequency content in $s\left(t\right)$. Paragraph 6.1, reference a provides information on ASD processing of stationary time traces including normalized random and bias error estimates. For analysis filter bandwidth B_e in Hz, and averaging time T in seconds, the normalized random error for the ASD estimate is given by

$$\varepsilon_r\left[\hat{G}_{xx}\left(f\right)\right] \approx \frac{1}{\sqrt{B_e T}} \tag{17}$$

while the normalized bias error is given by

$$\varepsilon_b\left[\hat{G}_{xx}\left(f_r\right)\right] = \frac{B_r}{B_e}\tan^{-1}\left(\frac{B_e}{B_r}\right) - 1 \tag{18}$$

where

$$B_r \approx 2\zeta f_r$$

is an estimate of the half-power bandwidth of a resonant peak.

An ESD estimate is typically a scaled periodogram, scaled by multiplying the periodogram by the duration of the time trace $N\Delta t$, over a very short transient time trace that cannot be characterized by an ASD estimate. A uniform or end tapered uniform time window is generally placed over the significant portion of the time trace. For transient TWR time traces, ESD estimates are useful for comparing *r(t)* and *c(t)* in addition to examining the character of *s(t)*.

E7: SRS – Shock Response Spectra (global)

As in the case of the frequency domain estimates in E6 a comparison between SRS estimates for deterministic *r(t)* and stochastic *c(t)* can be made. The SRS estimate for the error time trace *s(t)* is related to an SRS estimate for pre-shock and post-shock considered to be random in nature (see Method 516).The SRS may be expressed as a time domain convolution of an impulse response function that has the character of the response to base-input of the mass of a single-degree-of-freedom mechanical system, with a certain percentage of critical damping. The SRS estimate is a function of the output of the mass displacement, velocity, and acceleration. If the maximum absolute acceleration (positive or negative) is selected over the time interval of excitation, and plotted versus the undamped natural frequency of the single-degree-of-freedom system, the resulting plot over a selected set of frequencies is referred to as a maximax shock response spectrum. It is becoming increasingly evident that for most cases of mechanical shock the pseudo-velocity SRS estimate is a more indicative measure of potential for mechanical damage (because mechanical damage is related to mechanical stress that, in turn, is proportional to relative velocity of a mass-spring-damper system). Various references provide the details of SRS computation. For transient time trace TWR comparison, the SRS of *r(t)* and *c(t)* is useful and demonstrates the faithfulness of shock reproduction under TWR. Computing the SRS for *s(t)* is less useful and difficult to interpret since random variable S should represent a noise source but not Normal distributed. The mathematics for the SRS computation over a transient $x(t)$ for $0 \le t \le T_r$ is given as follows:

$$SRS(f_n,\zeta) = \Im\left[y(t,f_n,\zeta)\right] = \Im\left[\int_0^T h_{f_n,\zeta}(t-\tau)\,x(\tau)\,d\tau\right] \text{ for } 0 \le T_r \le T$$

where,

$SRS(f_n)$ - the magnitude of the SRS at natural frequency f_n

$\Im$ - a nonlinear functional operating on the resulting convolution $y(t,f_n,\zeta)$

$h_{f_n,\zeta}(t-\tau)$ - impulse function response for a damped single-degree-of-freedom system with base input and undamped natural frequency f_n having damping ratio ζ.

$x(\tau)$ - finite input record $0 \le t \le T_r$

T - time of response assessment where generally $T_r < T$

Natural frequency, f_n, can extend beyond the sampling frequency of *x(t)*. The SRS estimate is computed through filtering a transient time record, and does not have a clear random error or bias error criterion. Numerically, the time trace sample rate should be ten times the bandwidth of the time trace in order to provide an acceptable error in the estimates (approximately 5 percent error).

4. REPLICATION ERROR TOLERANCE SPECIFICATION.

From the analyst point of view it is highly desirable to attempt to apply each of the expressions in paragraph 3 to assess the replication error. However, when it comes to TWR test tolerance specification only a few of these expressions can be easily interpreted after application. For example, requiring *s(t)* to be zero mean Gaussian with a specified standard deviation as a fraction of the peak values in *r(t)*, for a test to be within tolerance is unrealistic. Requiring correlation between *r(t)* and *s(t)* to be a set value e.g., 0.975, is likewise not practical nor meaningful. The TWR test tolerance specifications below should be easily interpreted and reflect the descriptive convenience of the expressions in paragraph 3. Generally for post-analysis processing to determine test tolerance compliance it is highly desirable that replication error tolerance specifications be tailored to the form of the time history being replicated and formally agreed to before testing. The varied form of *r(t)*, i.e., stationary, nonstationary, shock, Gaussian, non-Gaussian or any combination of all of these, requires replication error tolerance specification to be tailored based upon the form of *r(t)*. such tolerance specification is complicated by the fact that almost assuredly some form of windowing and averaging will need to be applied for which random and bias processing errors are not easily determined to be nominal. It is usually unclear as to the reference for the specification and if multiple references need to be provided as a function of the form of *r(t)*. In this case then there may be multiple replication error assessments and subsequent tolerance specifications.

For the suggested replication error test tolerances it is assumed that the measure of *r(t)* is a form of general amplitude "rms" level derived by computing the "average energy" of *r(t)* in terms of units-squared and then taking the square-root of this value. For Time Domain Moments this relates to the "root-energy-amplitude" except the rms duration of *r(t)* becomes the time averaging factor. For well defined transient vibration forms of *r(t)* or forms of *r(t)* for which root-mean-square duration is meaningful it is suggested that the reference of the specification be the "root-energy-amplitude". For the tolerance specifications proposed below the reference "root-energy-amplitude" (REA) is provided by the following expression:

$$REA = \sqrt{\frac{1}{T}\int_0^T r^2(t)\,dt} \leftrightarrow \sqrt{\frac{1}{N}\sum_{i=1}^{N} r^2(t_i)}$$

where removal of the overall mean of *r(t)* before computing REA is left to the form of *r(t)* and discretion of the analyst. This is a very general root-mean-square *r(t)* signal level and for multiple test tolerance specifications may be applied over segments of *r(t)*. (Other possible reference scaling, for example, might be the reference range which is generally very sensitive to outliers.)

There are five general categories of replication error tolerance specifications proposed here:

The first category relates directly to *s(t)* and is referenced for convenience to the overall "root-mean-square" level of *r(t)* defined as REA above. Of the two specifications the root-mean-square error is the most significant.

Category I. The mean error, for which the STA is estimated for the oversample time interval factor on *r(t)*, shall not exceed more than 1% of the rms amplitude of *r(t),* REA, over more than 5% (or 0.95 quantile) of the duration of *r(t)*.

The root-mean-square error, for which the STA is estimated for the oversample time interval factor on *r(t),* shall not exceed more than 10% of the rms amplitude of *r(t)* , REA, over more than 5% (or 0.95 quantile) of the time.

The second category relates to (1) stationary random portions of *r(t)*, (2) a periodogram estimate i.e., ESD, over *r(t)* or (3) some combination of (1) and (2). For Fourier based processing of *r(t)* and *c(t)* an ASD, a periodogram or an ESD estimate is assumed available for *r(t)* and *c(t)*. This includes stationary random vibration – Gaussian or non-Gaussian and shock specified in terms of an ESD estimate.

Category II. For portions of frequency domain the replication error related to the ASD or periodogram (ESD) shall not exceed the tolerance limits proposed for stationary random vibration when deterministic *r(t)* is considered the reference (see Method 514).

For the third category whereby a "Product Model" may be fit to *r(t)* of the form of a transient vibration then it is assumed that the analysis has defined *r(t)* in terms of a PM with a time domain rms estimate and an appropriately scaled normalized ASD estimate.

Category III. For the frequency domain portion of the PM, tolerance specification according to the Category II will apply. For the time domain portion of the PM tolerance specification according to Category I will apply.

The fourth category relates directly or *r(t)* as the form of a "shock" for which SRS estimates provide the most meaningful information.

Category IV. For shock the tolerance specification shall be in accord with that in Method 516. That is the tolerance specification shall not exceed the tolerance proposed for the SRS in Method 516 where deterministic *r(t)* is considered the reference against *c(t)*

The fifth category is very general and is based upon the FOT probability distribution as applied to the error *s(t)*. FOT is able to quantify the time for which the error is at or above a specified quantile level.

Category V. The 5th and 95th quantile of the FOT related to *s(t)* (for which no STA has been computed) shall not exceed more than 10% of the plus and minus rms amplitude of *r(t)* .